

1993

Futures markets in an open economy

Jae-Gyeong Kim
Iowa State University

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Futures markets in an open economy

by

Jae-Gyeong Kim

A Dissertation Submitted to the
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Iowa State University
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1993

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CHAPTER 1. INTRODUCTION

Everyone is confronted by the uncertainty or risk which comes from a lack of knowledge as to what the state of the world is going to be. Knight has differentiated between uncertainty and risk: risk if the individual feels able to attach probabilities to the various possible states of the world, uncertainty if the individual feels unable to do so. However, many economists use uncertainty and risk interchangeably. The fact that uncertainty or risk affects the actions and decisions of economic agents makes it an important topic for economic analysis. One way to manage this risk is the introduction of instruments such as futures contracts and option contracts to the market. Such contracts allow individuals to trade away or reduce uncertainty in return for a market price.

Increased trading activities and expanding markets in recent years and related economic or political shocks have created higher and more volatile prices resulting in greater uncertainty in decision making. This environment has led to an increased interest in the use of futures markets. Many markets are now much more international in nature than just a decade ago.

Foreign currencies have become some of the most actively traded commodities in the world and their markets promise to

grow in the foreseeable future. The power behind this activity, of course, is the growth of international trade and investment. Since the break down of the Bretton Woods fixed exchange rate regime in the early 1970s, foreign exchange rates have fluctuated wildly. So exchange rate risk has become an important issue for management of firms with international transactions. Commodity traders have been simultaneously confronted with increased price and exchange rate risk as they have engaged in international commodity trading. Consequently, commodity futures and currency futures markets or currency forward markets have rapidly developed and actively utilized in recent years as a way of efficiently coping with exchange rate and price risks.

A futures contract, which is a legally binding commitment to make or take delivery at a later date, of a fixed amount of a specific grade or quality of a commodity at a specific price, is used by hedgers to manage risk, and by speculators and arbitragers to earn trading profits. Futures trading serves a number of important functions vital to the health of a market economy through the role of hedgers, speculators and arbitragers. Two of the major economic functions of futures markets are risk-shifting and price discovery. Risk shifting, which is known as hedging, is the major economic justification for futures markets. It is the act of taking a position in the futures market that is opposite to one's cash market position

to protect the cash position against loss due to price fluctuations. Secondly, the futures price for a commodity represents most information about the future cash price. If this price serves as a signal to guide production, consumption, and financing decisions, it follows that the resultant allocation of resources will be more efficient than would be the case in the absence of markets where this information is made known. This price discovery role is a side benefit of futures trading.

The purpose of this dissertation is to analyze the competitive firm's behavior under the price and exchange rate uncertainty when the firm participates in futures markets to hedge against these risks. In the present study, firms are assumed to buy or sell the product domestically, produce the product as well as export or import the product from abroad. This study will provide some insight into the risk shifting role of futures markets and marketing strategy between domestic and foreign markets when there is both commodity price and exchange rate uncertainty. Also the firm's behavior is analyzed when basis risk, which is random fluctuations in the difference between the cash market price and the futures contract price at a specified date, is introduced to the model.

The volume of trade in forward markets has a significant level of activity relative to that in the futures markets. The

kinds of commodities which are actively traded on both forward and futures markets are generally international commodities (Paroush and Wolf). There are differences in analyzing the model depending upon whether traders export directly or export forward. Therefore, this dissertation will examine the hedging and production rules for the firm which exports forward and supplies output in the domestic market under price, exchange rate, and basis risk.

Many offshore traders use the U.S. futures markets as a risk management tool. Offshore hedgers are usually confronted with price and exchange rate risk. Exchange rate risk is important to offshore decision makers in that currency values can change between the time a futures contract is placed and the time the hedge is lifted. Additionally they face basis risk because trading commodities are not deliverable against contracts specified on futures exchanges or the delivery date of the futures contract does not coincide with the importing or exporting date of the output. Therefore, the offshore firm's behavior is analyzed under exchange rate and basis risk when they use the U.S. futures markets as a risk-management tool.

The empirical section will study an offshore firm as represented by a Korean importing firm. When a Korean firm imports products as inputs for final goods, it usually faces commodity price and exchange rate uncertainties. This

importing firm's concern is how it could import products with less price uncertainty. The way to do is to use U.S. commodity futures market. In Korea the forward exchange rate market is well developed, but futures markets do not exist. Korean importing firms usually import agricultural products such as corn, soybeans and wheat from the U.S. because these commodities are little produced or not produced in Korea. This type of agricultural product has severe price fluctuations relative to industrial products because of production uncertainty. These products are used as inputs for the production of final goods. Because of that, uncertainty in these input prices may make the supply and price of final goods uncertain. The management of commodity price risk by using futures markets could increase the social benefit of Korea. The purpose of the empirical study is to show with real data how futures markets and forward markets hedge a price and exchange rate risk, and what is the optimal hedge ratio. To do the empirical work, we choose corn as example.

This dissertation considers the three kinds of firm. The first firm buys or sells the product domestically, produces the product as well as export or import the product from abroad. The second firm is different from the first firm in that it exports or imports by forward contracts which are certain. The third firm is the importing firm in offshore country. Generally the purpose of this dissertation is to

analyze the behavior of above firms under the price and exchange rate uncertainties when futures markets or forward markets are available.

To accomplish the objectives, we will use following methods. First, we set up the model in an expected utility frame work. Second, we solve the model using Stein's theorem and the Kuhn Tucker condition method, and interpret the results. Finally, in the empirical study, a unit root test is used to test whether the stationary condition for each series is satisfied or not. We must use the series satisfied the stationary condition in order to do empirical work or get variance, covariance and correlation coefficients.

The plan of this dissertation is as follows. A literature review on futures markets is given in Chapter 2. Chapter 3 considers a general producing and trading firm's behavior in international trade with futures markets in the commodity and currency under price and exchange rate uncertainty. The first section is an introduction while the second and third sections involve a model with complete markets or incomplete markets, respectively. Subsequent sections reexamine the above model by adding basis risk.

In Chapter 4, a model is considered for a general producing and trading firm with a futures market in the commodity and currency under basis and exchange rate uncertainty. However, the firm's behavior is different in that

the firm exports or imports by forward contract.

Chapter 5 deals with the importing firm's behavior in offshore markets when a commodity futures market and a currency forward market is available. Chapter 6 is an empirical study of optimal hedging behavior in offshore markets by the importing firm considered in Chapter 5. Chapter 7 presents a short summary of the results and conclusions.

CHAPTER 2. LITERATURE REVIEW

Mckinnon (1967) is the first to consider hedging from the point of view of primary or agricultural producers. He distinguishes the farmer from the grain merchant holding inventories in that the farmer faces his own output uncertainty as well as output price uncertainty, but the grain merchant has only the latter problem. He shows how hedging can minimize risk under the conditions of both price and production uncertainty. However, in his model, planned production is not a decision variable, which is in contrast to the standard firm theory. He also adds the further income stabilizing possibility of buffer stocks. He concludes that hedging is an effective method in managing risk and that buffer stocks can be used to moderate production variability.

Holthausen (1979) incorporates forward-futures trading opportunities into the theory of the firm making decisions under the conditions of uncertainty. His major finding is that the firm which does not face production uncertainty will produce a level of output which depends only on the forward price and is, in particular, independent of the firm's degree of risk aversion and the probability distribution of the uncertain price. This phenomenon of independence of production and hedging or production and risk aversion is called

separation. Also, if firms are characterized by nonincreasing absolute risk aversion, the optimal hedge increases as the riskiness of the price uncertainty increases. He adds two additional results that more risk averse decision makers make larger hedges and that the existence of a forward-futures market results in a large output due to risk response.

Similarly, Feder, Just and Schmitz (FJS) (1980) examine the behavior of a competitive firm under price uncertainty where a futures market exists for the commodity produced by the firm. They find that the level of output is determined independently from the future spot price's probability distribution when futures market trading is allowed. The production decision is solely a function of the forward-futures price and costs of production. That is, with the presence of a futures market, a complete separation is maintained between the production decision and the futures trading decisions. The subjective distribution of futures spot price affects only the firm's involvement in futures trading. Conditions are then determined under which a firm will either hedge, speculate by buying futures contracts, or speculate by selling futures contracts. They indicate the important social benefit derived from the existence of a futures market because introducing futures markets will eliminate output fluctuations. However, in Holthausen and FJS, if production uncertainty or basis risk is introduced to their model, the

separation between production and hedging decision does not hold.

Batlin (1983) explores a model of a producer who faces output price uncertainty with imperfect time hedging opportunities in the futures market. That is, he extends Holthausen and FJS's model by adding basis risk. He demonstrates that the separation theorem between production and hedging doesn't hold except in the very special case of a perfect time hedge (i.e., the delivery date of the futures contract coincides with the marketing date of the output). Moreover, hedging actually exacerbates the effects that price expectations, risk, and risk aversion would have had on output in the absence of futures markets. He shows that higher expectations about future spot price risk generally reduce the level of forward sales, and a higher correlation between spot and futures price changes generally induces higher levels of output and short hedging in comparative static results.

Grant (1985) examines expected utility maximizing farmers facing just price or both price risk and quantity risk. A farmer facing joint price and output uncertainty behaves in a similar fashion to one confronting only price uncertainty when there is no chance to participate in forward trading. However, when forward markets exist, farmers behave differently depending upon whether they confront price risk only or joint price and quantity risk. If there is only price uncertainty,

the risk associated with the farmer's production can be eliminated completely through forward trading, and the separation theorem holds. When both price and quantity uncertainty are present, forward contracting will not eliminate all risk and the separation theorem does not hold. An optimal level of production and an optimal forward position depend upon the joint distribution of price and quantity and upon the farmer's degree of risk aversion. These relationships can be identified for special cases. Grant argues that these cases should be examined empirically in future research.

Benninga, Eldor and Zilcha (BEZ) (1985) derive optimal hedging and production rules for an exporting firm which faces both commodity-price and foreign exchange rate uncertainty. In their model, the hedging problem for an exporting firm differs from that generally considered in the literature because the exporting firm faces two tiers of uncertainty. When unbiased forward (or futures) markets for the commodity produced by the firm and for the foreign currency jointly exist, and when the two markets are independent, then optimization by the firm implies full hedging in both markets. The size of the commodity hedge is independent of the properties of the foreign exchange markets. However, the optimal foreign exchange hedge depends on the commodity hedge and the properties of the commodity forward markets. In addition if both forward markets exist, the firm's optimal production

level is independent of the firm's degree of risk aversion and the probability distribution of the uncertain prices. The effect of the absence of one or both of the forward markets on production depends on the consumption beta (a covariance term between the marginal utility of wealth and random prices, which expresses the riskiness of the unhedgeable risks) of the unhedgeable risks.

Kawai and Zilcha (KZ) (1986) examine the optimum behavior of a risk-averse international trading firm under exchange rate and commodity price uncertainties when forward exchange and commodity futures contracts are available. First they verify the Separation Theorem and the Full Double Hedging Theorem. The Separation Theorem states that the optimum output supply (for export) or input demand (for import) of the commodity is determined only by the trader's production technology and the product of the forward exchange rate and the commodity futures prices. Attitudes toward risk and the stochastic nature of the random variables determine the trader's involvements in forward foreign exchange and commodity futures. The Full Double Hedging Theorem states that the optimal policy is a perfect (full) hedge in the commodity and currency markets. The paper proves that the joint unbiasedness of the forward foreign exchange and commodity futures markets is sufficient for full double hedging to be optimal. Second, they investigate the

implications of the existence of both a forward foreign exchange rate market and a commodity futures markets in comparison to the case where only one (or no) market is available to the firm.

The introducing of a missing market(s) that satisfies joint unbiasedness will unambiguously increase the level of export. When the markets are biased against the exporter in the sense that he must pay a risk premium, the chance of securing a favorable effect (i.e., increased production for export) by establishing an additional market will be diminished, but remains high as long as the bias (or risk premium) is not too large. It is also demonstrated that making separately unbiased markets available does not automatically stimulate the level of international trade. In their model, if production uncertainty or basis risk is included (that is, more general, and actual) optimality conditions must be modified and are more complicated. In such a case, even a full double hedge will not diversify away risk all together. In BEG and KZ if basis risk is considered, then the separation theorem does not hold.

Thompson and Bond (1987) examine the optimal hedging behavior of offshore commodity traders under uncertainty of commodity prices and exchange rates. First, the standard commodity hedging framework is extended to incorporate exchange rate uncertainty, and second, to forward cover

transactions in the foreign exchange market, which offers the opportunity to reduce currency risk. The theoretical derivations indicate that exchange rate risk may affect commodity hedging decisions in situations where exchange rates and commodity prices are perceived to interact with one another over time. Also, the theory suggests that the extent of utilization of U.S. futures markets by offshore traders is influenced in part by their strategies for coping with exchange rate uncertainty. Examination of these theoretical propositions for Australian and U.S. wheat traders making hedging decisions on the Chicago futures exchange confirms that the magnitude of exchange rate effects can be substantial.

Fung and Lai (1991) develop a model under the assumption of stochastic prices to explain different hedging behaviors of a multinational firm. The hedging decision rules depend on the covariability of the uncertain export earnings (output price) and the exchange rate for the exporting (importing) firm. Furthermore, the production decision is no longer separable from exchange rate risk (that is, the separation theorem breaks down). They show that speculative activity in the forward market has a positive interaction with the level of export, and exchange rate uncertainty has a substantial impact on exports. That is, if the correlation coefficient between price and exchange rate, r , is non-negative, then the exchange

rate risk will have an adverse effect on trade. However, if r is very negative, the firm tends to increase its exports because the uncertain exchange rate risk helps the multinational firm reduce its total risk. Similar properties are also shown to hold for firms that import an input from abroad and are faced with both output price and exchange rate uncertainty.

Zilch and Eldor (1991) consider a model with a competitive risk-averse exporting firm who faces uncertain exchange rates in a multiperiod analysis. The capital stock (or fixed input) has to be determined at the outset while the variable input (labor) is chosen optimally at the beginning of each period, but before the realization of the exchange rate. The widely recognized separation result does not hold in their model even though the production function is deterministic. They show that introducing unbiased currency forward markets decreases the capital/labor ratio in all periods, and compared to the one-period case such a firm tends to overhedge, which is considered as a hedge against uncertain returns in the second period. This has some policy implications. For example, in the U.S. for tax purposes, one is considered as a speculator if one sells short in the forward market but is not long in the commodity (currency) itself. Their result shows that selling forward more than one's output (or its value) may stem from hedging purposes in a many-period model. In some

cases the introduction of unbiased forward markets results in higher investments and production in all dates.

In Mckinnon, Holthausen, FJS, Batlin and Grant the firm produces only for domestic use, and in BEZ, KZ, TB, FL and ZE the firm produces only for export. The above models do not consider a competitive firm which produces for export and domestic use in two-period framework. Many firms sell their products in both domestic and foreign markets. They also buy and sell in different market. The model of this dissertation will consider simultaneously domestic and foreign commodity markets and firm which can sell and buy in both markets. For example, when the foreign price is expected to be very high, if the firm can buy its product in the domestic market and sell to the foreign market, then the firm can obtain more profits. Therefore, this model is more general and provides some insight into the hedging role of futures markets on production and marketing strategy between domestic and foreign markets.

CHAPTER 3. A MODEL OF A PRODUCING AND TRADING FIRM FACING PRICE UNCERTAINTY IN INTERNATIONAL MARKETS

3.1. Introduction

The basic model considers the problem of a competitive, risk-averse firm which produces for export and domestic use in a two-period framework. The firm can also buy and sell in both domestic and foreign markets. The firm faces three kinds of uncertainty. First, the spot foreign exchange rate is a random variable r . Second, the domestic price of the commodity is a random variable p . Third, the firm faces a random variable s which is the foreign price of the commodity in the foreign currency. For simplicity we assume that the firm does not face other types of uncertainties, is a price taker and has identified the subjective probability distribution of all random variables. The firm has a Von Neumann-Morgenstern utility function $U(\pi)$ with the properties $U'(\pi) > 0$ and $U''(\pi) < 0$, and a cost function $C(Q)$ with $C'(Q) > 0$ and $C''(Q) \geq 0$, and maximizes the expected utility of its domestic-currency profits. The firm has access to both foreign exchange futures and commodity futures contracting in the domestic currency. We call this the case of complete markets, while the situation where the firm does not have access to both contracting opportunities is called the case of

incomplete markets.

3.2. The complete market model

In complete markets, the firm's profits in domestic currency units can be expressed as

$$\pi = \tilde{r}\tilde{s} Q_f + \tilde{p} Q_d - C(Q_f+Q_d) - K Q_f + (F - \tilde{p})Z + (R - \tilde{r})X$$

where a tilde ($\tilde{}$) denotes a random variable, and π , r , p and s are always random variables even when there is no tilde. The following notation is used:

\tilde{r} = spot foreign exchange rate in period (t+1)

\tilde{s} = the foreign price of the commodity in foreign currency in period (t+1)

Q_f = quantity exported (if $Q_f > 0$) or imported (if $Q_f < 0$) in period (t+1)

\tilde{p} = domestic price in period (t+1)

Q_d = quantity sold (if $Q_d > 0$) or purchased (if $Q_d < 0$) in domestic market in period (t+1)

$C(Q_f+Q_d)$ = cost function with $C'(Q) > 0$ and $C''(Q) \geq 0$

K = transaction cost (include transportation cost) of foreign trading per unit

F = futures price in t for delivery in period (t+1) (certain)

Z = the amount of the commodity sold (if $Z > 0$) or

purchased (if $Z < 0$) in the futures market

R = foreign exchange futures price in period t (certain)

\tilde{r} = foreign exchange futures price in period $(t+1)$

X = the amount of foreign exchange sold (if $X > 0$) or purchased (if $X < 0$) in the futures market.

There are four decision variables: Q_f , Q_d , Z and X , and three sources of uncertainty: \tilde{r} , \tilde{s} and \tilde{p} . While the efficient market hypothesis would imply that $\tilde{r}\tilde{s}$ and \tilde{p} should be related, they are viewed in a general fashion at this point. Also notice that the producer can choose to purchase in one market and sell in the other market depending on relative prices. The cost function is defined over net production and does not include the cost of transportation and storage. Thus, for example, if $Q_f = -100$, and $Q_d = 200$, the cost function is defined for the net production and sales of 100. If the firm is an international commodity trading company without production, then $C(Q)$ must be interpreted in a different fashion to represent the cost of purchasing, storing and transporting the commodity.

The optimization problem is

$$\text{Max}_{Q_f, Q_d, Z, X} \quad EU(\pi)$$

$$\text{s.t. } \pi = \tilde{r}\tilde{s}Q_f + \tilde{p}Q_d - C(Q_f+Q_d) - KQ_f + (F-\tilde{p})Z + (R-\tilde{r})X$$

There are four possible cases to be considered as far as

trade restrictions.

1. Q_f and Q_d unrestricted
2. $Q_f \geq 0$ and Q_d unrestricted (no foreign purchase)
3. Q_f unrestricted and $Q_d \geq 0$ (no domestic purchase)
4. $Q_f \geq 0$ and $Q_d \geq 0$ (no purchasing)

3.2.1. Case 1. Q_f and Q_d are unrestricted

In this case the competitive firm can export or import its output as well as sell or buy in domestic markets.

The first order conditions for an optimum are

$$E[U'(\pi) (\tilde{r}\tilde{s} - C' - K)] = 0 \quad (3.1)$$

$$E[U'(\pi) (\tilde{p} - C')] = 0 \quad (3.2)$$

$$E[U'(\pi) (F - \tilde{p})] = 0 \quad (3.3)$$

$$E[U'(\pi) (R - \tilde{r})] = 0 \quad (3.4)$$

Equation (1.2) can be rearranged to obtain

$$E[U'(\pi) \tilde{p}] = E[U'(\pi) C']$$

Equation (1.3) can be rearranged to obtain

$$E[U'(\pi) F] = E[U'(\pi) \tilde{p}]$$

Combining these expressions gives

$$F = C'(Q) \quad (3.5)$$

Since F and $C'(Q)$ are nonstochastic.

Under the condition of complete markets, the firm's production is given by equation (3.5). That is, the firm's output level is chosen at a point where marginal cost (C') is equal to the certain futures price. Hence the production decision does not depend on the utility function (i.e., the degree of risk aversion) or the probability distribution of the random variables. The intuition behind this result is that when production is not stochastic, the existence of both futures markets completely eliminates all risks for the firm since it can buy and sell freely in either market. This separation theorem was first proved by Danthine (1978) for a competitive firm [see also Kawai and Zilcha(1986)]. Its extension to our model demonstrates that the separation theorem still holds when the firm produces for export and domestic use under price and exchange rate uncertainty as long as it can freely trade in both markets. If commodity futures markets are highly competitive, we might expect F to be close to $E\tilde{p}$ (unbiasedness). Under this unbiasedness assumption, the risk averse firm will produce amounts close to the output levels that would be optimal if $E\tilde{p}$ were certain to occur.

To gain intuition about why production is always decided at the point that $F = C'(Q)$ under the condition of complete markets, suppose the expected foreign price is very high relative to the expected domestic price. The firm would choose

to purchase the commodity in the domestic market and sell it at the higher foreign price. The transaction levels would be determined by the relative prices, risk characteristic and the probability distributions of random variables. Given the opportunity to purchase the commodity domestically, the firm may not choose to produce at all. If the firm does produce, production will be equal to the excess of Q_f over $(-Q_d)$. This amount can be perfectly hedged on the domestic market given the standard separation result. Or consider the case where the expected domestic price is high relative to the expected foreign price. The firm may choose not to produce for the foreign market but rather purchase the commodity in the foreign market. In the domestic market it will produce enough to supply the excess of Q_d over $(-Q_f)$. This amount can be hedged using the domestic futures market, and separation holds. Therefore, in any cases, separation holds.

If we use the relation that when two random variables (X and Y) are not independent, $E(XY) = EX EY + \text{cov}(X,Y)$, then we can get the following equations by rewriting equations (3.1), (3.3) and (3.4)

$$E[U'(\pi)(C'+K)] = EU'(\pi) E(\tilde{r}\tilde{s}) + \text{cov}(U'(\pi), r\tilde{s}) \quad (3.6)$$

$$EU'(\pi) F = EU'(\pi) E\tilde{p} + \text{cov}(U'(\pi), p) \quad (3.7)$$

$$EU'(\pi) R = EU'(\pi) E\tilde{r} + \text{cov}(U'(\pi), r) \quad (3.8)$$

Combining equation (3.6) with (3.5) gives

$$F = E\tilde{r}s - K + \text{cov}(U'(\pi), rs) / EU'(\pi) \quad (3.9)$$

Equation (3.7) can be used to obtain

$$F = E\tilde{p} + \text{cov}(U'(\pi), p) / EU'(\pi) \quad (3.10)$$

Equation (3.9) and (3.10) can be used to give meaning to the idea of the certainty equivalent. The relationships between the certainty equivalent, the expected risky return, and the risk premium are expressed as follows:

$$\text{Certainty Equivalent} = \text{Expected Risky Return} - \text{Risk Premium}$$

In this model, the expected prices will not lead to the same level of production as F since they differ by the covariance term.

Consider now some assumptions about the price distributions. Assume that \tilde{p} and \tilde{r} are statistically independent. This assumes independence between domestic prices and the exchange rate. Then $\text{cov}(p, r) = 0$. Stein's theorem (1973), derived independently by Rubinstein (1976) states that if two random variable X and Y are jointly normally distributed, and g is a differentiable function, then $\text{cov}(g(X), Y) = E[g'(X)] \text{cov}(X, Y)$. Stein's theorem is especially useful in expected utility models in which covariance terms between marginal utility of profit and random

variables appear in the profit equation. Applying the theorem to the covariance terms in the equations (3.6), (3.7) and (3.8) leads to the following expressions.

$$\begin{aligned} \text{cov}(U'(\pi), rs) &= EU''(\pi) \text{cov}(\pi, rs) \\ &= EU''(\pi) [Q_f \text{var}(rs) + Q_p \text{cov}(rs, p) \\ &\quad - Z \text{cov}(rs, p) - X \text{cov}(rs, r)] \end{aligned} \quad (3.11)$$

$$\begin{aligned} \text{cov}(U'(\pi), p) &= EU''(\pi) \text{cov}(\pi, p) \\ &= EU''(\pi) [Q_f \text{cov}(rs, p) + Q_p \text{var}(p) - Z \text{var}(p)] \end{aligned} \quad (3.12)$$

$$\text{cov}(U'(\pi), r) = EU''(\pi) [Q_f \text{cov}(rs, r) - X \text{var}(r)] \quad (3.13)$$

where $E(\cdot)$, $\text{var}(\cdot)$ and $\text{cov}(\cdot)$ are the expectation, variance and covariance operators. Substituting (3.11), (3.12) and (3.13) into (3.6), (3.7) and (3.8), and dividing by $EU'(\pi)$ gives the following expression

$$\begin{aligned} E(\tilde{r}\tilde{s}) - C' - K &= \lambda [Q_f \text{var}(rs) + Q_p \text{cov}(rs, p) - \\ &\quad Z \text{cov}(rs, p) - X \text{cov}(rs, r)] \end{aligned} \quad (3.14)$$

$$E\tilde{p} - F = \lambda [Q_f \text{cov}(rs, p) + Q_p \text{var}(p) - Z \text{var}(p)] \quad (3.15)$$

$$E\tilde{r} - R = \lambda [Q_f \text{cov}(rs, r) - X \text{var}(r)] \quad (3.16)$$

where $-[EU''(\pi)]/[EU'(\pi)] \equiv \lambda > 0$ represents the producer's Pratt-Arrow coefficient of absolute risk aversion. If futures markets are considered unbiased (i.e., $F = E\tilde{p}$ and $R = E\tilde{r}$),

then the equation (3.15) and (3.16) will give

$$Z = Q_D + Q_F \text{cov}(rs, p) / \text{var}(p) \quad (3.17)$$

$$X = Q_F [\text{cov}(rs, r) / \text{var}(r)] \quad (3.18)$$

Substituting (3.17) and (3.18) into (3.14) yields

$$Q_F^* = \frac{[E\tilde{I}\tilde{S} - K - F]}{\lambda \text{var}(rs) [1 - \rho_{rsp}^2 - \rho_{rsr}^2]} \quad (3.19)$$

where $\rho_{rsp}^2 = \text{cov}^2(rs, p) / \text{var}(rs) \text{var}(p)$

$\rho_{rsr}^2 = \text{cov}^2(rs, r) / \text{var}(rs) \text{var}(r)$.

Here $\text{var}(rs) [1 - \rho_{rsp}^2 - \rho_{rsr}^2] = \text{var}(s) [E\tilde{r}^2 - \bar{r}^2 \rho_{sp}^2] > 0$

because $E\tilde{r}^2 - \bar{r}^2 = \text{var}(r)$ and $0 < \rho_{sp}^2 \leq 1$.

Furthermore, $\text{var}(rs) = \bar{r}^2 \text{var}(s) + \bar{s}^2 \text{var}(r) + \text{var}(r) \text{var}(s)$
 $= \bar{s}^2 \text{var}(r) + E\tilde{r}^2 \text{var}(s)$.

We can get Q_D^* from equations (3.5) and (3.19) since $Q = Q_F + Q_D$

$$Q_D^* = Q^* - Q_F^*$$

Equation (3.17) indicates the optimal hedge. The hedging behavior of the firm will depend on its attitude toward risk and the distribution of random variables. If $Q_F^* = 0$, the firm can hedge perfectly against the price risk. When $Q_F^* > 0$, whether there is overhedging or underhedging depends on the

ratio between $\text{cov}(r_s, p)$ and $\text{var}(p)$. Usually under the assumption of unbiasedness and no basis risk, the firm will hedge its entire output. However, in this model (when both foreign and domestic markets are considered) the firm will not hedge its entire output even with unbiasedness and no basis risk since the price uncertainty is multiplied by the exchange rate uncertainty. That is, the firm may overhedge or underhedge even with the unbiasedness and no basis risk assumptions.

Equation (3.19) represents the optimal export or import quantity. Although the firm hedges using commodity and currency futures contracts, there is a still uncertainty in exporting or importing decision because the firm's exporting or importing decision is affected by the distribution of random variables and risk attitude. Hence in determining the absolute level of export or import, the firm takes into account its attitude toward risk, the probability distribution of random variables and correlations between random variables.

When the net expected foreign price in terms of domestic currency unit ($E\tilde{r}_s - K$) is greater than the futures contract price (F) or marginal cost (C') the firm exports its output in foreign market. If $(E\tilde{r}_s - K) \leq F$ then the firm doesn't export. If the net expected foreign price equals the certain futures contract price, the firm will want to sell all its output in the domestic market and not import any units. This is because

there is no uncertainty in the domestic market due to the perfect hedge available in the domestic market. However, if $(E\tilde{r}_s - K) < F$, then the firm will import the product and sell it in the domestic market and in this case the optimal hedge: $Z = Q_D - Q_F \text{ cov}(r_s, p) / \text{var}(p)$. Therefore, only when there is a benefit covering the uncertainty in foreign market will the firm want to sell its output in the foreign market.

Consider now another case. If the expected foreign price is greater than the expected domestic price, then $Q_D < 0$ and $Q_F > 0$ with no restrictions on Q_D and Q_F . If the firm wants to hedge the price risks of its output, then the firm buys commodity futures contracts ($-Q_D$) in order to hedge the price risk for purchasing Q_D in the domestic market and sells commodity futures contracts ($Q_F \text{ cov}(r_s, p) / \text{var}(p)$) in order to hedge the foreign price risk for exporting Q_F in the foreign market, and simultaneously sells currency futures contracts ($E\tilde{s} Q_F$) in order to hedge the exchange risk for exporting Q_F . Then the optimal amount of futures contracts:

$$Z = -Q_D + Q_F \text{ cov}(r_s, f) / \text{var}(p)$$

Equation (3.18) representing the optimal currency futures position can be expressed as

$$X = E(\tilde{s}) Q_F^* \quad (3.20)$$

because $\text{cov}(rs, r) = E(\tilde{s}) \text{var}(r)$ due to $\text{cov}(r, s) = 0$

Then we say that the firm fully hedges in the currency market; namely, it sells all its foreign exchange proceeds.

Now suppose that the commodity futures price incorporates a risk premium, so that $F < E\tilde{p}$. Then $\text{cov}(U'(\pi), p)$ in equation (3.7) must be negative since $EU'(\pi) > 0$. In order for the covariance to be negative, the value of U' must decline when \tilde{p} rises, which (since U is concave) means that π must rise when \tilde{p} rises. The right hand side term in equation (3.10) must be negative. That is,

$$EU''(\pi) [Q_f \text{cov}(rs, p) + Q_d \text{var}(p) - Z \text{var}(p)] < 0$$

Then we can get

$$Z < Q_d + Q_f \text{cov}(rs, p) / \text{var}(p)$$

If the commodity futures price incorporate a risk premium the firm hedges less than in the case of unbiasedness.

Suppose that the exchange rate futures market incorporates a risk premium, so that $R < E\tilde{r}$. Then $\text{cov}(U'(\pi), r)$ in equation (3.8) must be negative. From equation (3.11)

$$EU''(\pi) [Q_f \text{cov}(rs, r) - X \text{var}(r)] < 0$$

$$X < Q_f \text{cov}(rs, r) / \text{var}(r)$$

$$\text{or } X < E\tilde{s} Q_f$$

Therefore

$$X < E\tilde{s} Q_f \quad \text{iff} \quad R < E\tilde{r}$$

That is, if the exchange futures market incorporates a risk premium, the firm hedges less than in the case of unbiasedness in the currency futures market since risk premium is considered as a hedging cost. Hence, we can imagine that there will be underhedging under normal backwardation (the futures price is below the expected spot price) and overhedging under contango (the futures price exceeds the expected spot price).

3.2.2. Case 2. $Q_f \geq 0$ and Q_D is unrestricted

Consider the case of the restrictions on imports but no restrictions on domestic sales Q_D . There are three possible scenarios.

- i) First, a firm sells its output both domestically and abroad when the expected foreign price and the expected domestic price are competitive.
- ii) Second, when the expected foreign price is relatively higher than the expected domestic price, the firm buys its output in domestic market and sells it in foreign market. That is, all product produced and bought in domestic market is sold in foreign market.
- iii) Third, when the expected domestic price is relatively

higher than the expected foreign price, the firm sells all output domestically produced in the domestic market because of the restriction on imports. Here we can imagine that tariff restrictions make imports impossible. Therefore, the first and second scenarios are not actually restricted but the third is restricted.

The need for inequality constraints requires the use of Kuhn-Tucker condition for solving the problem. The restriction on Q_f (≥ 0) changes equation (3.1) to the inequality condition.

$$E[U'(\pi)(\tilde{r}\tilde{s} - C' - K)] \leq 0 \quad \text{and} \quad [\partial EU(\pi)/\partial Q_f] Q_f = 0 \quad (3.1')$$

Equation (3.2) (3.3) and (3.4) are the same as before.

i) In this restricted form if $Q_f > 0$, all results are the same as the unrestricted case because $E[U'(\pi)(\tilde{r}\tilde{s} - C' - K)] = 0$. That is, the firm sells its output both domestically and abroad because the expected foreign price and the expected domestic price are competitive. So the first scenario is included in this case.

ii) If $Q_f > 0$ and Q_D is unrestricted, the inequality first order condition is changed to the equality condition. Because $Q_f > 0$ means that the expected foreign price is greater than the expected domestic price, the second scenario is also included in this case.

iii) If $Q_f = 0$, implying that the restriction is binding and

there may be an incentive to import, equation (3.1') reduces to the following

$$E\tilde{r}\tilde{s} - K - F + \text{cov}(U'(\pi), rs) / EU'(\pi) \leq 0$$

using the fact that

$$E(U'(\pi) \tilde{r}\tilde{s}) = EU'(\pi)E\tilde{r}\tilde{s} + \text{cov}(U'(\pi), rs)$$

$$\text{and } F = C'(Q).$$

If Q_f is zero, then the above expression could be strictly negative which would imply that decreasing Q_f (below zero) would raise profits. For example, if the producer could buy cheap foreign goods ($(E\tilde{r}\tilde{s} - K) < F$) expected utility would be higher. Given the constraint, however, he can only sell in the foreign market. Given no incentive to do so, separation still holds since equations (3.2) and (3.3) imply

$$F = C'(Q)$$

$$\text{and } Q^* = Q_D^* = Z$$

Therefore, the firm's production is still decided at the point that $F = C'(Q)$. The firm doesn't export its output and sells its entire output in domestic market. It also hedges its entire output. This occurs because the domestic market can be perfectly hedged.

3.2.3. Case 3. Q_f is unrestricted and $Q_d \geq 0$

In this part we consider that the case when the firm can not buy in the domestic market but can buy or sell in the foreign market. The restriction is in the domestic market but not in the foreign market. If domestic market conditions are favorable, the firm's domestic supply will be greater than zero, but if conditions are not favorable domestic supply will be zero. In case 3 there are three possible scenarios.

- i) First, when the expected foreign price and expected domestic price are competitive the firm sells its output in the domestic and foreign market. This is exactly the same as the first scenario in case 2.
- ii) Second, when the expected domestic price is relatively higher than the expected foreign price, the firm buys its output in foreign market and sells it in domestic market. That is, all product produced domestically and bought in the foreign market is sold in the domestic market.
- iii) Third, when the expected foreign price is relatively higher than the expected domestic price, the firm sells all output produced in domestic market to the foreign market but can not buy output in domestic market due to some restrictions.

It is difficult to imagine the third scenario in the real

world. Possible cases might be a large rice producer who only exports the product, or a high expected domestic transaction cost due to some expected strikes. Equation (3.2) is changed as following due to a restriction on Q_0

$$E[U'(\pi)(p - C'(Q))] \leq 0 \quad \text{and} \quad [\partial EU(\pi)/\partial Q_0] Q_0 = 0 \quad (3.2')$$

Equation (3.1) (3.3) and (3.4) are same.

i) If domestic supply is greater than zero, then all results are the same as the unrestricted case. That is, interior solutions take place and separation holds. That is, if sales take place in the domestic market, separation will hold.

The first and second scenarios are applicable to above case.

ii) Since if $Q_0 > 0$ means that the expected domestic price is relatively higher than the expected foreign price, the second scenario is also applicable to this case.

iii) If $Q_0 = 0$, equation (3.2') can be rerearranged to obtain $[EU'(\pi) \tilde{p} \leq EU'(\pi) C'(Q)]$ and equation (3.3) can be rearranged to obtain $[EU'(\pi) F = EU'(\pi) \tilde{p}]$. Combining these two expressions gives

$$F \leq C'(Q) \quad \Leftrightarrow \quad Q_0 = 0$$

That is, when domestic supply is zero, the futures price is

less than the marginal cost, or if the futures price is less than the marginal cost, then the firm does not get involved in domestic trading.

From equations (3.17) and (3.18)

$$Z = Q_f \text{ cov}(rs, p) / \text{var}(p) \quad (3.17')$$

$$X = Q_f \text{ cov}(rs, r) / \text{var}(r) \quad (3.18')$$

Q_f has the same formula as in equation (3.19).

$$Q^* = Q_f^* \quad \text{because } Q_0 = 0 \quad \text{in } [Q = Q_f + Q_0]$$

When $Q_0 = 0$, the production decision is based on equation (3.19). However, the separation theorem doesn't hold. That is, the firm's production decision is affected by the probability distributions of random variables and risk attitudes because firm's production is based on the foreign market conditions and foreign market uncertainty can not be completely eliminated by the futures markets. The above case is similar to the third scenario in case 2, but separation does not hold.

3.2.4. Case 4. $Q_f \geq 0$ and $Q_0 \geq 0$

Consider cases with restrictions on the export and domestic supply. There are four possible situations.

i) $Q_f > 0$ and $Q_0 > 0$

ii) $Q_f > 0$ and $Q_0 = 0$

$$\text{iii) } Q_f = 0 \text{ and } Q_D > 0$$

$$\text{iv) } Q_f = 0 \text{ and } Q_D = 0$$

The restrictions of $Q_f \geq 0$ and $Q_D \geq 0$ give the following first order conditions:

$$E[U'(\pi)(\tilde{r}\tilde{s} - C' - K)] \leq 0 \text{ and } [\partial EU(\pi)/\partial Q_f] Q_f = 0 \quad (3.1'')$$

$$E[U'(\pi)(\tilde{p} - C'(Q))] \leq 0 \text{ and } [\partial EU(\pi)/\partial Q_D] Q_D = 0 \quad (3.2'')$$

Equation (3.3) and (3.4) are the same as in the previous case.

i) If $Q_f > 0$, equation (3.1'') holds with equality, while (3.2'') may not. Then equation (3.1'') and (3.2'') using (3.3) would be as follows:

$$EU'(\pi) E \tilde{r}\tilde{s} + \text{cov}(U'(\pi), rs) - EU'(\pi)(C' + K) = 0 \quad (3.1''')$$

$$EU'(\pi) F - EU'(\pi) C'(Q) \leq 0 \quad (3.2''')$$

Subtracting (3.1''') from (3.2''') and dividing by $EU'(\pi)$ gives:

$$F + K - E\tilde{r}\tilde{s} - \text{cov}(U'(\pi), rs) / EU'(\pi) \leq 0$$

Because the third term in above inequality equation represents the positive risk premium, we get the following result

$$F < [E\tilde{r}\tilde{s} - K] \Leftrightarrow Q_f > 0 \quad (3.21)$$

and from equation (3.1''') using equation (3.17') and (3.18')

$$Q_F^* = \frac{E\tilde{r}\tilde{s} - K - C'(Q)}{\lambda \text{var}(rs) (1 - \rho_{rsp}^2 - \rho_{rsr}^2)} \quad (3.22)$$

In this case we can not make the production decision or determine the level of domestic supply because of the simultaneous binding conditions in the two markets. In order to determine the production decision we need to use a constraint such that

$Q_D > 0$ or $Q_D = 0$. If $Q_F > 0$ and $Q_D > 0$, then all results are the same as the unrestricted case because a firm sells its output domestically and abroad only when the net expected foreign price and the expected domestic price are competitive. Alternatively if $Q_D > 0$, we need a constraint on Q_F such as $Q_F > 0$ or $Q_F = 0$ to solve the system.

ii) If $Q_F > 0$ and $Q_D = 0$ (this is the same as the third scenario in case 3),

$$Z = Q_F \text{cov}(rs, p) / \text{var}(p)$$

$$X = Q_F \text{cov}(rs, r) / \text{var}(r)$$

and equation (3.1''') holds with equality, while (3.2''') still may not hold. Then from equation (3.2''') we can get following $F \leq C'(Q) \rightarrow Q_D$ and from equation (3.22) $E\tilde{r}\tilde{s} - K - C'(Q) > 0$ in order for $Q_F^* > 0$. Combining the

above two inequality conditions gives

$$F \leq C'(Q) < (E\tilde{r}\tilde{s} - K) \quad \Leftrightarrow \quad Q_f > 0 \text{ and } Q_d = 0$$

Q_f^* is the same as in equation (3.22)

$$Q^* = Q_f^* \text{ because } Q_d \text{ is set to zero by } Q = Q_f + Q_d$$

Therefore, if $Q_f > 0$ and $Q_d = 0$, production decision is based on equation (3.22) because the production decision is based on the foreign market conditions due to a restriction on the domestic market ($Q_d = 0$). The separation theorem does not hold. This is the same as the case of the unrestricted Q_f and $Q_d = 0$.

iii) If $Q_d > 0$, equation (3.2'') holds with equality but (3.1'') may not. Then equation (3.1'') and (3.2'') using (3.3) would be as follows:

$$EU'(\pi) E\tilde{r}\tilde{s} + \text{cov}(U'(\pi), rs) - EU'(\pi) (C' + K) \leq 0 \quad (3.1^a)$$

$$EU'(\pi) F - EU'(\pi) C'(Q) = 0 \quad (3.2^a)$$

Subtracting (3.2^a) from (3.1^a) gives:

$$E\tilde{r}\tilde{s} - F - K \leq -\text{cov}(U'(\pi), rs) / EU'(\pi) \Leftrightarrow Q_d > 0 \quad (3.3^a)$$

Here, $-\text{cov}(U'(\pi), rs) / EU'(\pi)$ represents the risk premium which is positive. Therefore, if the difference between

expected foreign price and certain futures price is less than the risk premium, then the firm will supply its output in domestic market. In this case, we can decide the production decision. The reason is that we can get the condition $[F = C'(Q)]$ from equation (3.2'') with equality and (3.3) since equation (3.2'') holds with equality. That is, when $Q_D > 0$, the possible situation is $Q_F > 0$ or $Q_F = 0$. If $Q_D > 0$ and $Q_F > 0$, this is the same as the unrestricted case. If $Q_D > 0$ and $Q_F = 0$, the firm produces output only if the condition $(F = C'(Q))$ holds because the firm sells output in domestic market. If the one condition $(Q_D > 0)$ exists, the following equations are the same as the unrestricted case.

$$\begin{aligned} F &= C'(Q) \\ Z &= Q_D + Q_F \text{cov}(rs, p) / \text{var}(p) \\ X &= Q_F \text{cov}(rs, r) / \text{var}(r) \end{aligned}$$

If $Q_D > 0$ and $Q_F = 0$ (this is the same as the third scenario in Case 2)

$$\begin{aligned} F &= C'(Q) \\ Z &= Q^* = Q_D \end{aligned}$$

Whenever $Q_F = 0$, $X = 0$ because X is a hedge on exchange rate uncertainty. From equation (3.3^a) we can get following result

$$F \geq E \tilde{r}_s \quad \leftrightarrow \quad Q_D > 0 \quad \text{and} \quad Q_F = 0$$

Therefore, if $Q_D > 0$ and $Q_F = 0$, production is decided at the point that $F = C'(Q)$ and separation theorem still holds and optimal hedge ratio is 1.

iv) If $Q_F = Q_D = 0$ (only speculative terms exist)

$$Z = (F - E \tilde{p}) / \lambda \text{ var}(p)$$

$$X = (R - E \tilde{r}) / \lambda \text{ var}(r)$$

If $F > E \tilde{p}$ then $Z > 0$: sell the commodity futures.

If $F < E \tilde{p}$ then $Z < 0$: purchase the commodity futures.

If $F = E \tilde{p}$ (unbiased), then $Z = 0$

In this case a firm acts as a speculator who, according to Anderson and Danthine (1983), is a participant in the futures market but does not possess the any quantity of physical commodity. If the futures price is an unbiased estimate of the expected spot price, the firm does not want to participate in the futures market. Because the firm acts as a speculator it requires a risk premium for its risk-bearing in the futures markets. Therefore, the firm participates in the futures markets only when there is normal backwardation or contango. The firm sells the commodity or currency futures under the contango, which is defined as the futures price that is greater than the expected spot price ($F > E \tilde{p}$ or $R > E \tilde{r}$).

The firm purchases the commodity or currency futures under the normal backwardation, which is the futures price that is less than the expected spot price ($F < E\tilde{p}$ or $R < E\tilde{r}$). Therefore, if the restrictions on the two markets ($Q_f = Q_d = 0$) exist, the firm does not produce and participate in the futures market as a speculator only when there is normal backwardation or contango.

3.3. The incomplete market model

In this section we consider interactions between the production and futures markets available to the firm for hedging. Denote the optimal solutions in the absence of the currency futures market by π^c and in the absence of the commodity futures market by π^e and in the absence of the commodity and currency futures market by π^n , as opposed to π^* , the optimal solution when both hedging markets exist. All variables are the same as in the complete market model.

3.3.1. No commodity and currency futures markets

Consider the case where the firm has access to neither a commodity futures market nor an exchange rate futures market. The firm cannot avoid risks through futures hedging. The optimization problem is

$$\begin{aligned} \text{Max} \quad & EU(\pi^n) \\ \text{s.t} \quad & \pi^n = \tilde{r}\tilde{s}Q_F + \tilde{p}Q_D - C(Q_F+Q_D) - K Q_F \end{aligned} \quad (3.23)$$

The first order conditions for optimality are

$$E[U'(\pi^n) (\tilde{r}\tilde{s} - C' - K)] = 0 \quad (3.24)$$

$$E[U'(\pi^n) (\tilde{p} - C')] = 0 \quad (3.25)$$

Rewriting (2.24) and (2.25) yield

$$EU'(\pi^n) C' = EU'(\pi^n) E \tilde{r}\tilde{s} + \text{cov}(U'(\pi^n), rs) \quad (3.26)$$

$$EU'(\pi^n) C' = EU'(\pi^n) E\tilde{p} + \text{cov}(U'(\pi^n), p) \quad (3.27)$$

If we solve equation (3.26) and (3.27) simultaneously we obtain the following solutions.

$$Q_F^n = \frac{\text{var}(p) [E\tilde{r}\tilde{s} - C'] - \text{cov}(rs, p) [E\tilde{p} - C']}{\lambda \text{var}(rs) \text{var}(p) (1 - \rho_{rsp}^2)} \quad (3.28)$$

$$Q_D^n = \frac{\text{var}(rs) [E\tilde{p} - C'] - \text{cov}(rs, p) [E\tilde{r}\tilde{s} - C']}{\lambda \text{var}(rs) \text{var}(p) (1 - \rho_{rsp}^2)} \quad (3.29)$$

$$Q_F^n \geq 0 \quad \text{iff} \quad \frac{[E\tilde{r}\tilde{s} - C']}{[E\tilde{p} - C']} \geq \frac{\text{cov}(rs, p)}{\text{var}(p)}$$

$$Q_D^n \geq 0 \quad \text{iff} \quad \frac{[E\tilde{p} - C']}{[E\tilde{r}\tilde{s} - C']} \geq \frac{\text{cov}(rs, p)}{\text{var}(rs)}$$

The firm can not hedge its risk in the domestic or foreign market because there are no futures markets. When the ratio of expected net unit revenue from the foreign market and the domestic market is greater than the ratio between the covariance of r_s and p , and variance of p , the firm exports its output in the foreign market. If $\text{cov}(r_s, p) < \text{var}(p)$, the firm is willing to export its output even though expected net unit revenue in the foreign market is less than the expected net unit revenue in the domestic market. If we compare equation (3.28) with equation (3.19), whether to export or import depends only on foreign market conditions when both hedging markets exist, but depends on both foreign and domestic market conditions when the futures markets do not exist. If the ratio of expected net unit revenue between the domestic market and the foreign market is greater than the ratio of the covariance between r_s and p , and the variance of r_s , the firm supplies its output in the domestic market.

Now investigate the impact of the existence of a futures market on production. When both hedging markets exist, $C'(Q^*) = F$ (equation 3.5). When neither hedging markets exists, from equation (3.27) we can get the following:

$$EU'(\pi^n)C' = EU'(\pi^n)E\tilde{p} + \text{cov}(U'(\pi^n), p).$$

If divide by $EU'(\pi^n)$, then

$$C'(Q^n) = E\tilde{p} + \text{cov}(U'(\pi^n), p)/EU'(\pi^n) \quad (3.27')$$

If the two equations (3.5) and (3.27') are compared

$$\frac{C'(Q^*)}{C'(Q^n)} - \frac{F}{\tilde{E}p + \frac{\text{cov}(U'(\Pi), p)}{EU'(\pi)}} > 1 \quad (3.30)$$

If we assume that futures market is considered unbiased (i.e., $\tilde{E}p=F$), then the left hand side of equation (3.30) is greater than 1. The reason is that $\tilde{E}p= F$ and $\text{cov}(U'(\pi^n), p) / EU'(\pi^n) = [EU''(\pi^n) / EU'(\pi^n)] \text{cov}(\pi, p) < 0$ since $EU''(\pi^n) < 0$, $EU'(\pi^n) > 0$ and $\text{cov}(\pi, p) > 0$. If both futures markets do not exist, the firm can not hedge some price risks. This makes production less than in the certainty case. Because production is less under uncertainty than under certainty (Sandmo). Therefore, if futures markets are introduced to the market, production increases since the producer (or hedger) can make a production decision with certainty by using futures markets.

3.3.2. A commodity futures markets only

Consider the case where the firm only has access to a commodity futures market. The optimization problem is

$$\begin{aligned} \text{Max } & EU(\pi^c) \\ \text{s.t } & \pi^c = \tilde{r}\tilde{s} Q_f + \tilde{p}Q_0 - C(Q_f+Q_0) - K Q_f + (F - \tilde{p}) Z \end{aligned} \quad (3.31)$$

The first order conditions for optimality are

$$E[U'(\pi^c)(\tilde{r}\tilde{s} - C' - K)] = 0 \quad (3.32)$$

$$E[U'(\pi^c)(\tilde{p} - C')] = 0 \quad (3.33)$$

$$E[U'(\pi^c)(F - \tilde{p})] = 0 \quad (3.34)$$

From (3.33) and (3.34) we can get

$$F = C'(Q^c) \quad (3.35)$$

With the existence of a commodity futures market only the firm's production is given by equation (3.35). Hence Q^c does not depend on the utility function or the probability distribution of the random variables. That means the separation theorem still holds when only the commodity futures market is introduced to the market.

If futures market is considered unbiased (i.e., $F = E\tilde{p}$), then we get the following equations.

$$Q_F^c = \frac{E\tilde{r}\tilde{s} - K - F}{\lambda \text{var}(rs) (1 - \rho_{rsp}^2)} \quad (3.36)$$

$$Z_D^c = Q_D^c + Q_F^c \frac{\text{cov}(rs, p)}{\text{var}(p)} \quad (3.37)$$

$$Q_D^c = Q^c - Q_F^c \quad (3.38)$$

If we compare equation (3.36) with equation (3.17) the condition in which the firm exports or imports its output is exactly the same as the case of the existence of the commodity and currency futures market although the absolute level of export is different. That means, currency futures market do not affect the condition of export but affect the absolute amount of export. Equation (3.37) is exactly the same as equation (3.17) which represents the optimal hedge under the existence of both hedging markets.

From equation (3.17) and (3.36)

$$Q_f^* \geq Q_f^c \quad \text{since} \quad \rho_{rsr}^2 \geq 0$$

Because the correlation between r_s and r is positive, export under the existence of two futures markets is greater than the export under the existence of the commodity futures market only.

3.3.3. An exchange futures market only

Consider the case where there is only an exchange rate futures markets. The optimization problem is

$$\begin{aligned} \text{Max} \quad & EU(\pi^e) \\ \text{s.t} \quad & \pi^e = \tilde{r}_s Q_f + \tilde{p} Q_D - C(Q_f + Q_D) - K Q_f + (R - \tilde{r}) X \end{aligned} \quad (3.39)$$

The first order conditions for optimality are

$$E[U'(\pi^e) \{ \tilde{r}\tilde{s} - C' - K \}] = 0 \quad (3.40)$$

$$E[U'(\pi^e) \{ \tilde{p} - C' \}] = 0 \quad (3.41)$$

$$E[U'(\pi^e) \{ R - \tilde{r} \}] = 0 \quad (3.42)$$

Rewriting equations (3.40), (3.41) and (3.42) yield

$$EU'(\pi) E\tilde{r}\tilde{s} + \text{cov}(U'(\pi), rs) - EU'(\pi) [C' + K] = 0$$

$$EU'(\pi) E\tilde{p} + \text{cov}(U'(\pi), p) - EU'(\pi) C' = 0$$

$$EU'(\pi) R - EU'(\pi) E\tilde{r} - \text{cov}(U'(\pi), r) = 0$$

Dividing by $EU'(\pi)$ and applying Stein's theorem to the above equations gives:

$$E\tilde{r}\tilde{s} - (C' + K) = \lambda [Q_F \text{var}(rs) + Q_D \text{cov}(rs, p) - X \text{cov}(rs, r)]$$

$$E\tilde{p} - C' = \lambda [Q_F \text{cov}(rs, p) + Q_D \text{var}(p)]$$

$$E\tilde{r} - R = \lambda [Q_F \text{cov}(rs, r) - X \text{var}(r)]$$

If we simultaneously solve the above equations, then we get the following expressions:

$$Q_F^e = \frac{\text{var}(p) [E\tilde{r}\tilde{s} - C' - K] - \text{cov}(rs, p) [E\tilde{p} - C']}{\lambda \text{var}(rs) \text{var}(p) [1 - \rho_{rsp}^2 - \rho_{rsr}^2]} \quad (3.43)$$

$$Q_D^e = \frac{[E\tilde{p} - C'] \text{var}(rs) (1 - \rho_{rsr}^2) - [E\tilde{r}\tilde{s} - C' - K] \text{cov}(rs, p)}{\lambda \text{var}(rs) \text{var}(p) [1 - \rho_{rsp}^2 - \rho_{rsr}^2]} \quad (3.44)$$

$$Q_F^* \geq 0 \text{ iff } \frac{[E\tilde{r}\tilde{S} - C' - K]}{[E\tilde{p} - C']} \geq \frac{\text{cov}(rs, p)}{\text{var}(p)}$$

$$Q_D^o \geq 0 \text{ iff } \frac{[E\tilde{p} - C']}{[E\tilde{r}\tilde{S} - C' - K]} \geq \frac{\text{cov}(rs, p)}{\text{var}(rs) (1 - \rho_{rsr}^2)}$$

When there is only an exchange futures market, whether the firm exports or not depends on the relation between the ratio of net expected returns and the ratio of covariance and variance of random variables (rs and p). If we compare equation (3.43) with (3.28) we can see that currency futures market does not affect the condition of export but does affect the absolute amount of export. The condition of domestic market supply is different from the case without both hedging markets.

3.4. The model with basis risk in commodity futures

In this section we add basis risk in the commodity futures contract to the previous complete market model. The firm additionally faces a random variable \tilde{f} which is the commodity futures price at maturity date. Since basis risk is random fluctuations in the difference between the cash market price and the futures contract price at a specified date, this risk can not be eliminated by hedging.

The optimization problem is

$$\text{Max } EU(\pi)$$

$$\text{s.t } \pi = \tilde{r}\tilde{s} Q_f + \tilde{p} Q_D - C(Q_f+Q_D) - K Q_f + (F - \tilde{f})Z + (R - \tilde{r})X$$

The first order conditions are

$$E[U'(\pi)(\tilde{r}\tilde{s} - C' - K)] = 0 \quad (3.45)$$

$$E[U'(\pi)(\tilde{p} - C')] = 0 \quad (3.46)$$

$$E[U'(\pi)(F - \tilde{f})] = 0 \quad (3.47)$$

$$E[U'(\pi)(R - \tilde{r})] = 0 \quad (3.48)$$

Applying Stein's theorem to the first order conditions and rearranging them gives the following equations:

$$\begin{aligned} E\tilde{r}\tilde{s} - C'(Q) - K &= \lambda[Q_f \text{ var}(rs) + Q_D \text{ cov}(rs,p) - Z \text{ cov}(rs,f) \\ &\quad - X \text{ cov}(rs,r)] \end{aligned} \quad (3.49)$$

$$E\tilde{p} - C'(Q) = \lambda[Q_f \text{ cov}(rs,p) + Q_D \text{ var}(p) - Z \text{ cov}(p,f)] \quad (3.50)$$

$$E\tilde{f} - F = \lambda[Q_f \text{ cov}(rs,f) + Q_D \text{ cov}(p,f) - Z \text{ var}(f)] \quad (3.51)$$

$$E\tilde{r} - R = \lambda[Q_f \text{ cov}(rs,r) - X \text{ var}(r)] \quad (3.52)$$

If futures markets are considered unbiased (i.e., $F=E\tilde{p}$ and $R=E\tilde{r}$), then equations (3.51) and (3.52) yield

$$Z = Q_f \text{ cov}(rs,f)/\text{var}(f) + Q_D \text{ cov}(p,f)/\text{var}(f) \quad (3.53)$$

$$X = Q_f \text{ cov}(rs,r)/\text{var}(r) \quad (3.54)$$

Equation (3.53) represents the optimal hedge. The level of the hedge consists of two parts: the first part is for hedging the foreign price risk, second is for hedging the domestic price risk. The firm sells the commodity futures contract $(Q_D \text{ cov}(rs, f)/\text{var}(f))$ to hedge the domestic price risk of selling output (Q_D) in domestic market and sells the commodity futures contract $(Q_F \text{ cov}(rs, f)/\text{var}(f))$ to hedge the foreign price risk, and simultaneously sells the currency futures contracts $(Q_F \text{ cov}(rs, r)/\text{var}(r))$ in order to hedge the exchange rate risk for exporting Q_F .

In no markets (foreign and domestic) can the firm completely hedge the price risks, because foreign price is multiplied by exchange rate and basis risk exists in commodity futures markets. So the hedging effectiveness depends on the relationships between foreign price or domestic price and futures price.

If we simultaneously solve equations (3.49), (3.50), (3.51) and (3.52) we get following:

$$Q_F^* = \frac{S(1 - \rho_{pf}^2) \text{var}(p) - P(\rho_{rsp} - \rho_{pf}\rho_{rsf})\sqrt{\text{var}(rs)}\sqrt{\text{var}(p)}}{D} \quad (3.55)$$

$$Q_D^* = \frac{P(1 - \rho_{rsf}^2 - \rho_{rsr}^2) \text{var}(rs)}{D} - \frac{S(\rho_{rsp} - \rho_{pf}\rho_{rsf})\sqrt{\text{var}(rs)}\sqrt{\text{var}(p)}}{D} \quad (3.56)$$

$$Z^* = \frac{S(\rho_{rsf} - \rho_{rsp}\rho_{pf}) \text{var}(p) \sqrt{\text{var}}(rs) \sqrt{\text{var}}(f) - P \rho \text{var}(rs) \sqrt{\text{var}}(p) \sqrt{\text{var}}}{\text{var}(f) D} \quad (3.57)$$

$$X^* = Q_F^* \frac{\text{cov}(rs, r)}{\text{var}(r)} \quad (3.58)$$

where $D \equiv \lambda \text{var}(rs) \text{var}(p) [1 - \rho_{pf}^2 - \rho_{rsp}^2 - \rho_{rsf}^2 - \rho_{rsr}^2 + \rho_{rsr}^2 \rho_{pf}^2 + 2 \rho_{rsp} \rho_{rsf} \rho_{pf}]$ is denominator.

$S \equiv [E\tilde{r}s - C'(Q) - K]$ is the expected net revenue in foreign market

$P \equiv [E\tilde{p} - C'(Q)]$ is the expected net revenue in domestic market

$$\rho \equiv \rho_{rsr}^2 \rho_{pf} + \rho_{rsp} \rho_{rsf} - \rho_{pf}$$

Equations (3.55) and (3.56) represent the optimal export or import quantity, and optimal quantity of the domestic supply or purchase. Equations (3.57) and (3.58) indicate the optimal commodity and currency hedge.

If we assume that $D > 0$

$$Q_F^* \geq 0 \text{ iff } \frac{S}{P} \geq \frac{(\rho_{rsp} - \rho_{pf}\rho_{rsf}) \sqrt{\text{var}}(p)}{(1 - \rho_{pf}^2) \sqrt{\text{var}}(p)} \quad (3.59)$$

$$Q_D^* \geq 0 \text{ iff } \frac{P}{S} \geq \frac{(\rho_{rsp} - \rho_{pf}\rho_{rsf}) \sqrt{\text{var}}(p)}{(1 - \rho_{rsf}^2 - \rho_{rsr}^2) \sqrt{\text{var}}(rs)} \quad (3.60)$$

$$Z^* \geq 0 \text{ iff } \frac{S}{P} \geq \frac{\rho \sqrt{\text{var}}(IS)}{(\rho_{rsf} - \rho_{rsp} \rho_{pf}) \sqrt{\text{var}}(P)} \quad (3.61)$$

Equation (3.59) and (3.60) represent conditions on whether the firm exports or imports in foreign market and sells or buys in the domestic market. Equation (3.61) indicates whether the firm sells or buys futures contracts or does not involve itself in the futures market.

If basis risk in commodity futures market is introduced, the production and hedging decision are not separated, and the production decision depends on risk attitudes and/or the probability distribution of the random variables. Under the existence of basis risk, hedging means that price risk is substituted by basis risk. Hedging is effective because the price variance is usually larger than the basis variance. The export or import decision is affected by domestic and foreign market conditions when basis risk is introduced to the model. Alternatively, the export or import decision is affected only by foreign market conditions without basis risk. The firm can perfectly hedge its risk in domestic market with no basis risk, but the firm can overhedge or underhedge in the domestic market under basis risk even though the firm's hedges are the same in the foreign market regardless of basis risk or no basis risk.

3.5. Summary

The objective of this chapter examines behavior of an expected utility maximizing firm which faces both price and exchange rate uncertainty when the firm has access to both a commodity futures and a currency futures markets. To facilitate this examination in the complete market model, four possible cases are considered as far as trade restrictions.

First, when Q_f and Q_0 are unrestricted, the production decision does not depend on the utility function or the probability distribution of the random variables. That is, the separation theorem holds. However, marketing decisions (and hedging decision) are affected by uncertainty even though there are hedging instruments. Whether the firm exports or imports depends on the relative prices between the expected net foreign price ($E\tilde{r}\tilde{s} - k$) and the certain futures price (F). When $(E\tilde{r}\tilde{s} - k) > F$ or $(E\tilde{r}\tilde{s} - k) \gg F$, the firm exports and if $(E\tilde{r}\tilde{s} - k) \gg F$, its optimal hedge: $Z = -Q_0 + Q_f \text{cov}(rs, p) / \text{var}(p)$ and if $(E\tilde{r}\tilde{s} - k) > F$ (but not much large), its optimal hedge: $Z = Q_0 + Q_f \text{cov}(rs, p) / \text{var}(p)$. When $(E\tilde{r}\tilde{s} - k) = F$, the firm does not involve in foreign trading, and the optimal hedge $Z = Q_0$. When $(E\tilde{r}\tilde{s} - k) < F$, the firm imports and its optimal hedge $Z = Q_0 - Q_f \text{cov}(rs, p) / \text{var}(p)$. In a commodity market, the firm may overhedge or underhedge even with the unbiasedness and no basis risk assumptions. However, in a currency market,

the firm fully hedges. If the commodity or currency futures price incorporates a risk premium, the firm hedges less than in the case of unbiasedness. So we can imagine that there will be underhedging under normal backwardation and overhedging under contango.

Second, when $Q_f \geq 0$ and Q_D is unrestricted if $Q_f > 0$, all results are the same as the unrestricted case. If $Q_f = 0$, the separation theorem still holds, and the firm hedges its entire output.

Third, when Q_f is unrestricted and $Q_D \geq 0$ if $Q_D = 0$, the separation theorem does not hold because firm's production is based on the foreign market conditions with uncertainty, and the firm's hedge is not a full hedge, which depends on the relationships between covariance and variance of random variables.

Finally, when $Q_f \geq 0$ and $Q_D \geq 0$, there are four possible situations. However, because three situations are similar to the previous cases, we consider the case of $Q_f = Q_D = 0$. If $Q_f = Q_D = 0$, the firm does not produce and participate in the futures market as a speculator, and it participates in the futures markets only when there is normal backwardation or contango.

When we compare the complete market model with the incomplete market model, we find that if futures markets are introduced to the market, production increases since the

producer (or hedger) can make a production decision with certainty by using futures markets.

If basis risk in commodity futures market is introduced to the complete market model, the production and hedging decision are not separated, and the production decision depends on risk attitudes and/or the probability distribution of the random variables. The firm can overhedge or underhedge in the domestic market with basis risk even though the firm's hedges are the same in the foreign market regardless of basis risk or no basis risk.

This chapter is important because this study provides some insight into the risk shifting role of futures market and marketing strategy when there is both commodity and exchange rate uncertainty, and this model is applicable in real world if cost function is known. In next chapter we'll examine the general producing and trading firm with a futures market in the commodity and currency under basis and exchange rate uncertainty. However, the firm's behavior is different in that the firm exports or imports by forward contracts.

CHAPTER 4. THE BEHAVIOR OF A FORWARD EXPORTING OR IMPORTING FIRM

4.1. Introduction

Following Paroush and Wolf (1986), "the volume of trade in forward markets is a significant level of activity relative to that in the futures markets. Commodities which are actively traded on both forward and futures markets are generally international commodities, i.e., commodities that are traded on domestic markets as well as on foreign markets." Futures contracts and forward contracts are usually thought to be synonymous in most of the academic literature. But in general this is not true, although they serve the same economic functions. Forward contracts are distinguished from futures contracts by their differing legal characteristics and specifications. Following Black (1976), "a forward contract is a contract to buy or sell at a price that stays fixed for the life of the contract; a futures contract is settled every day and rewritten at the new futures price; a futures contract is like a series of forward contracts." There are possible disadvantages in forward contracts: first, there is the possibility of default, second, forward contracts are not traded continuously, third, there are high search costs and transaction costs due to illiquidity. The advantage of forward

contracts is that there is no basis risk.

There are differences in the competitive firm's behavior between when the firm exports directly and exports by forward contract under uncertainty. Therefore, we also need to examine the hedging and production rules for the firm which can export or import forward and supply or purchase output in the domestic market under price, and exchange rate uncertainties, and basis risk. The market environments are the same as in the model of Chapter 3.

4.2. The model

In this case the firm's profits in domestic currency units can be expressed as

$$\pi = \tilde{r} p_f Q_f + \tilde{p} Q_0 - C(Q_f + Q_0) - KQ_f + (F - \tilde{f})Z + (R - \tilde{r})X$$

The following notation is used:

p_f = the commodity forward contract price in foreign currency to be paid in (t+1) (certain)

Q_f = quantity exported forward (if $Q_f > 0$) or imported forward (if $Q_f < 0$) in (t+1)

Q_0 = quantity in domestic market in (t+1)

K = unit cost of forward trading (i.e., search cost and

transaction cost)

F = futures price in t for delivery in $t+1$ (certain)

\tilde{f} = futures price in $(t+1)$

All other variables are the same as in the previous model

There are four decision variables: Q_f , Q_d , Z and X , and three sources of uncertainty: \tilde{r} , \tilde{p} , \tilde{f} . The producer (or trader) can choose to purchase in one market and sell in the other market depending on relative prices. As in the model of Chapter 3, if the firm is an international commodity trading company without production, then $C(Q)$ must be interpreted in a different fashion to represent the cost of purchasing, storing and transporting the commodity.

The optimization problem is

$$\begin{aligned} \text{Max}_{Q_f, Q_d, Z, X} \quad & EU(\pi) \quad \text{s.t} \\ \pi = & \tilde{r}p_f Q_f + \tilde{p}Q_d - C(Q_f+Q_d) - KQ_f + (F-\tilde{f})Z + (R-\tilde{r})X \end{aligned}$$

There are four possible cases.

1. Q_f and Q_d are unrestricted
2. $Q_f \geq 0$ and Q_d is unrestricted (no foreign purchase)
3. Q_f unrestricted and $Q_d \geq 0$ (no domestic purchase)
4. $Q_f \geq 0$ and $Q_d \geq 0$ (no purchasing)

4.2.1. Case 1. Q_f and Q_d are unrestricted

The competitive firm can export or import forward as well as sell or buy its output in domestic markets in this case.

The first order conditions for an optimum are

$$E[U'(\pi) (\tilde{r}p_f - C' - K)] = 0 \quad (4.1)$$

$$E[U'(\pi) (\tilde{p} - C')] = 0 \quad (4.2)$$

$$E[U'(\pi) (F - \tilde{f})] = 0 \quad (4.3)$$

$$E[U'(\pi) (R - \tilde{r})] = 0 \quad (4.4)$$

Substituting (4.4) into (4.1) and simplifying yields

$$EU'(\pi) R p_f = E[U'(\pi) (C' + K)] \quad (4.5)$$

We can divide (4.5) by $EU'(\pi)$ since $R p_f$ and C' are deterministic. Then

$$[R p_f - K] = C'(Q) \quad (4.6)$$

Here $[R p_f - K]$ is a certain net unit revenue of forward trading in terms of domestic currency (net forward price). Under the existence of both commodity and currency futures markets, the firm's optimum production is given by equation (4.6). Hence Q^* ($=Q_f+Q_d$) does not depend on the utility

function (i.e., the degree of risk aversion) or the probability distribution of the random variables. Although basis risk due to a uncertain \tilde{f} exists, the separation theorem still holds since exchange rate uncertainty can be completely eliminated by the exchange rate futures market.

Why do the first order conditions differ between Chapter 3 and 4. The intuition is that because production under certainty is larger than under uncertainty (Sandmo), the firm want to decide the production decision under certainty. The firm's production decision in chapter 3 is decided at the domestic market condition because domestic market price uncertainty can be completely hedged by using domestic commodity futures market, but in the foreign market risk still exists because the foreign price multiplied by exchange rate is not completely hedged by futures markets. While the firm's production decision in this chapter is decided at the certain foreign market condition. Because the firm exports or imports by forward contract without uncertainty and exchange rate uncertainty can be completely hedged by currency futures market but domestic market uncertainty can not be completely hedged due to basis risk. Therefore, the first order conditions differ between Chapter 3 and 4 because of different market conditions and the firm's favor on certainty.

Combining equation (4.2) and (4.5) and rewriting (4.3) and (4.5) yields

$$(Rp_f - K) = E\tilde{p} + \text{cov}(U'(\pi), p) / EU'(\pi) \quad (4.7)$$

$$F = E\tilde{f} + \text{cov}(U'(\pi), f) / EU'(\pi) \quad (4.8)$$

$$R = E\tilde{r} + \text{cov}(U'(\pi), r) / EU'(\pi) \quad (4.9)$$

We assume that \tilde{p} (or \tilde{f}) and \tilde{r} are independent. Then $\text{cov}(p, r) = 0$ and $\text{cov}(f, r) = 0$. Applying Stein's theorem to equation (4.7) yields:

$$E\tilde{p} - (Rp_f - K) = \lambda [Q_D \text{var}(p) - Z \text{cov}(p, f)]$$

$$\text{If } E\tilde{p} > (Rp_f - K) \text{ then } Z / Q_D < \text{var}(p) / \text{cov}(p, f)$$

When the expected domestic price is greater than the net forward price regardless of biased or unbiasedness assumptions, the optimal hedge ratio depends on the relationships between the variance and covariance of random variables.

If we solve the above equations using Stein's theorem we get the following expressions

$$Z^* = Q_D \frac{\text{cov}(p, f)}{\text{var}(f)} - \frac{(E\tilde{f} - F)}{\lambda \text{var}(f)} \quad (4.10)$$

$$Q_D^* = \frac{E\tilde{p} + K - Rp_f}{\lambda \text{var}(p) (1 - \rho^2)} - \frac{(E\tilde{f} - F) \text{cov}(p, f)}{\lambda \text{var}(p) \text{var}(f) (1 - \rho^2)} \quad (4.11)$$

$$X^* = p_F Q_F - \frac{[E\tilde{f} - R]}{\lambda \text{var}(f)} \quad (4.12)$$

where $\rho^2 = [\{\text{cov}(p, f)\}^2] / [\text{var}(p) \text{var}(f)]$ represents the correlation coefficient. Theoretically, spot and futures prices should exhibit positive covariance and this relationship is confirmed empirically (Ederington (1979)). The first term in the solution (4.10) is the hedging component. The second term is the speculative component. If the futures market is considered unbiased (i.e., $F = E\tilde{f}$), then the speculative component disappears, and the optimal hedge ratio is

$$\frac{Z}{Q_D} = \frac{\text{cov}(p, f)}{\text{var}(f)} \quad (4.13)$$

The optimal hedge is not a full hedge since there is basis risk. If \tilde{p} and \tilde{f} are perfectly correlated, the optimal hedge is a full hedge. However, if the futures market is considered biased, then the futures position is altered by the amount of bias adjusted by the level of risk aversion and the futures price variability. Risk attitudes (λ) only affect the speculative component.

The first term in the solution (4.11) is the hedging component. The second term is the speculative component. If the futures market is considered unbiased (i.e., $F = E\tilde{f}$), then the speculative component disappears, and optimal domestic supply is

$$Q_D^* = \frac{E p + K - R p_F}{\lambda (1 - \rho^2) \text{var}(p)} \quad (4.14)$$

Although the firm hedges using the futures contract, there is still uncertainty. That is, price uncertainty cannot be completely eliminated due to basis risk. Hence, in determining the absolute level of domestic supply, the firm takes into account its attitude toward risk, the probability distribution of random variable \tilde{p} , futures prices and correlation between \tilde{p} and \tilde{f} .

If the futures price is an unbiased estimate of the expected futures price,

$$\begin{aligned} Q_D &\geq 0 && \text{iff } E\tilde{p} \geq R p_F - K \\ Q_D &< 0 && \text{iff } E\tilde{p} < R p_F - K \end{aligned}$$

That is, if expected domestic price is less than the net forward price, then the firm buy in domestic market and sell it foreign market by forward contracts.

Substituting (4.11) into (4.10) yields

$$Z^* = \frac{[E\tilde{p} + K - R p_F] \text{cov}(p, f)}{\lambda (1 - \rho^2) \text{var}(f) \text{var}(p)} - \frac{[E\tilde{f} - F]}{\lambda (1 - \rho^2) \text{var}(f)} \quad (4.15)$$

Under the unbiased condition ($F = E\tilde{f}$)

$$z^* = \frac{[\tilde{E}p + K - Rp_f] \text{cov}(p, f)}{\lambda(1-\rho^2) \text{var}(f) \text{var}(p)} \quad (4.15)$$

$$z^* \geq 0 \quad \text{iff} \quad \tilde{E}p \geq Rp_f - K$$

If we assume unbiasedness in commodity futures market and if $\tilde{E}p > [Rp_f - K]$, then a short position is taken in futures and a short spot position is taken in the domestic market. That is, the firm sells output in domestic market, and also hedges. However, if $\tilde{E}p = [Rp_f - K]$, the firm does not use the futures market, because the firm want to sell its output in the foreign market by forward contract without uncertainty. If $\tilde{E}p < [Rp_f - K]$, $Q_D < 0$, a long position is taken in futures and long spot position is taken in the domestic market. That is, the firm buys its output in domestic market and sells it in foreign market, and hedges its output bought in domestic market by purchasing futures. Therefore, the firm always hedges its output regardless of whether $\tilde{E}p > (Rp_f - K)$ or $\tilde{E}p < (Rp_f - K)$. That is, there is no speculation under the unbiasedness assumption. This is different from other model's results. Here, exchange rate uncertainty cannot affect commodity hedging decisions because domestic price uncertainty is hedged using the commodity futures contract and the exchange rate uncertainty is completely eliminated by using

the exchange rate futures contract.

The first and the second term in the right hand side of equation (4.12) are the hedging component and the speculative component. If the exchange futures market is considered unbiased (i.e., $R = E\tilde{r}$)

$$X^* = p_f Q_f \quad (4.16)$$

Then we say that the firm fully hedges; namely, it sells on the futures market all its foreign exchange proceeds. The sign of X^* depends on Q^* . If $Q_f > 0$, the firm sells currency futures contracts to hedge the exchange risk for exporting Q_f by forward contract. If $Q_f < 0$, the firm buys currency futures contracts to hedge the exchange risk for importing Q_f by forward contract. Therefore, if firm exports or imports forward and sells or buys output in the domestic market and basis risk is allowed, then the separation theorem still holds, and the firm fully hedges in currency market, but doesn't fully hedge in commodity market.

With and without basis risk

When there is no basis risk (i.e., $\tilde{f} = \tilde{p}$), the optimal level of output is

$$Q_D^* = Z^* + \frac{E\tilde{p} + K - Rp_F}{\lambda \text{var}(p)} \quad (4.17)$$

The relationship between Q_D and Z is therefore given by

$$Q_D^* \geq Z \quad \text{as} \quad E\tilde{p} \geq R p_f - K$$

It is clear that producers will completely hedge only if they expect $E\tilde{p} = (Rp_f - K)$. If the firm expects $E\tilde{p} > (R p_f - K)$, they will be less willing to buy insurance in the futures market. On the other hand, if the firm expects $E\tilde{p} < (R p_f - K)$, they will attempt to overhedge by selling more output in the futures markets than they plan to supply in the domestic market.

In the case of basis risk

$$Q_D^* = \frac{[E\tilde{p} + K - Rp_F]}{\lambda \text{var}(p)} + Z \frac{\text{cov}(p, f)}{\text{var}(p)} \quad (4.18)$$

It can be seen that the general relationship between Q_D^* and Z is

$$Q_D \geq Z \quad \text{as} \quad (E\tilde{p} + K - Rp_F) \geq \lambda \text{var}(p) \left(1 - \frac{\text{cov}(p, f)}{\text{var}(p)}\right) Z$$

Because $\lambda \text{var}(p) [Q_D - Z] =$

$$(E\tilde{p} + K - Rp_f) - \lambda \text{var}(p) \left\{1 - \frac{\text{cov}(p, f)}{\text{var}(p)}\right\} Z$$

The two models are equivalent only if $\beta (= \text{cov}(p, f) / \text{var}(p)) = 1$ (a sufficient condition for which is $\tilde{p} = \tilde{f}$). If $\Delta\tilde{f}$ is less volatile than $\Delta\tilde{p}$ ($\beta < 1$), the firm could overhedge by selling more output in futures market than they plan to supply output in domestic market, even though they expect $E\tilde{p} < (R p_f - K)$.

4.2.2. Case 2. $Q_f \geq 0$ and Q_d is unrestricted

Consider the case of restrictions on imports but no restrictions on domestic sales. The restriction on $Q_d (\geq)$ changes equation (4.1) to the inequality condition:

$$E[U'(\pi)(\tilde{r}p_f - C'(Q) - K) \leq 0 \quad \text{and} \quad [\partial EU(\pi) / \partial Q_f] Q_f = 0 \quad (4.1')$$

Equation (4.2) (4.3) and (4.4) are the same as before.

In this restricted form if $Q_f > 0$, all results are the same as the unrestricted case. In this restricted case there are two possible situations. First, $Q_f > 0$ and $Q_d > 0$ because the net hedged forward price $(R p_f - K)$ and expected domestic price $(E\tilde{p})$ are very competitive. Second, $Q_f > 0$ and $Q_d < 0$ because $(R p_f - K) \geq E\tilde{p}$. That is, all product produced and bought in domestic market and sold in foreign market. If $Q_f = 0$, from equation (4.1') and (4.4) we can get following:

$$R p_f - K \leq C'(Q) \quad \Leftrightarrow \quad Q_f = 0$$

If we rearrange the first order conditions, we can get the following solutions:

$$Q_D^* = \frac{E\tilde{p} - C'}{\lambda \text{var}(p) (1 - \rho_{pf}^2)}$$

$$Z^* = \frac{(E\tilde{p} - C') \text{cov}(p, f)}{\lambda \text{var}(p) (1 - \rho_{pf}^2)}$$

$$Z / Q_D = \text{cov}(p, f) / \text{var}(f)$$

If $(Rp_f - K) \leq C'(Q)$, then $Q_f = 0$ and the firm's production is decided in equation of Q_D^* . That is, separation between the production and marketing decisions does not hold and firm's production decision is affected by the probability of random variables and risk attitudes because the firm's production is based on the domestic market conditions and domestic market uncertainty cannot completely eliminated by the futures market due to basis risk. If $E\tilde{p} > C'(Q)$, a short position is taken in futures and a short spot position is taken in the domestic markets and the optimal hedge ratio $(Z/Q) = \text{cov}(p, f) / \text{var}(f)$. If $E\tilde{p} = C'(Q)$, no position is taken in futures and spot markets.

4.2.3. Case 3. Q_f is unrestricted and $Q_d \geq 0$

In this case the firm can not buy in the domestic market but can buy or sell in the foreign market. Equation (4.2) will be changed as following due to a restriction on Q_d .

$$E[U'(\pi)(\tilde{p} - C'(Q))] \leq 0 \quad \text{and} \quad [\partial EU(\pi)/\partial Q_d] Q_d = 0 \quad (4.2')$$

Equation (4.1) (4.3) and (4.4) are same.

If $Q_d > 0$, all results are the same as the unrestricted case.

However, if $Q_d = 0$, we can get different results:

$$E\tilde{p} \leq (Rp_f - K) \quad \Leftrightarrow \quad Q_d = 0$$

$$Rp_f - K = C'(Q)$$

$$Q^* = Q_f^* \quad \text{and} \quad Q_d = Z = 0$$

$$X = p_f Q_f$$

Therefore, if $E\tilde{p} \leq (Rp_f - K)$, then the firm's production decision is still decided at the point that $(Rp_f - K) = C'(Q)$, and the firm does not supply its output in the domestic market and export all its output by forward contract and does not take commodity futures contract. That is, separation theorem holds only under specific conditions: $E\tilde{p} \leq (Rp_f - K)$.

The firm fully hedges in foreign exchange market.

4.2.4. Case 4. when $Q_f \geq 0$ and $Q_d \geq 0$

Consider the case of the restrictions on the export and domestic supply. The restrictions of $Q_f \geq 0$ and $Q_d \geq 0$ bring the following first order conditions:

$$E[U'(\pi)(\tilde{r}\tilde{s} - C'(Q))] \leq 0 \quad \text{and} \quad [\partial EU(\pi)/\partial Q_f] Q_f = 0 \quad (4.1'')$$

$$E[U'(\pi)(\tilde{p} - C'(Q))] \leq 0 \quad \text{and} \quad [\partial EU(\pi)/\partial Q_d] Q_d = 0 \quad (4.2'')$$

Equation (4.3) and (4.4) are the same as the previous case.

i). If $Q_f > 0$, equation (4.1'') holds with equality while (4.2'') may not. Subtracting (4.1'') with equality from (4.2'') and using (4.4) gives the following condition:

$$E\tilde{p} - (Rp_f - K) \leq -\text{cov}(U'(\pi), p)/EU'(\pi) \Leftrightarrow Q_f > 0$$

Here $-\text{cov}(U'(\pi), p)/EU'(\pi)$ represents the risk premium which is positive. If the difference between certain net forward price and expected domestic spot price is less than the risk premium, the firm will export its output by forward contract.

If $Q_f > 0$ and $Q_d = 0$

$$Rp_f - K = C'(Q)$$

$$Q = Q_f \quad \text{and} \quad Q_d = Z = 0$$

$$X = p_f Q_f$$

If $Q_f > 0$ and $Q_D = 0$, production is decided at the point that $[Rp_f - K] = C'(Q)$, and all output will export by forward contract and no hedge in commodity markets.

ii). If $Q_D > 0$, equation (4.2'') holds with equality but (4.1'') may not. Subtracting (4.2'') with equality from (4.1'') gives:

$$E\tilde{p} > Rp_f - K \quad \Leftrightarrow \quad Q_D > 0$$

In this case we can not decide the production decision or the level of export. Because of that, we need another restriction such as $Q_f = 0$.

If $Q_D > 0$ and $Q_f = 0$,

$$Z = Q_D \text{ cov}(p, f) / \text{var}(f)$$

$$Q_D = (E\tilde{p} - C'(Q)) / \lambda \text{ var}(p) (1 - \rho_{pf}^2)$$

If $Q_D > 0$ and $Q_f = 0$, then $Rp_f - K \leq C'(Q)$ and other things are the same as above. That is, the production decision is decided in the equation Q_D . If $E\tilde{p} > C'(Q)$, a short position is taken in futures and short spot position is taken in the domestic market. However, the separation does not hold.

4.3. Summary

The objective of this chapter examines the hedging and production rules for the firm which can export or import forward and supply or purchase output in the domestic market under price, and exchange rate uncertainties, and basis risk. The main results of this chapter are as followings:

First, when Q_f and Q_d are unrestricted, the production and hedging decisions are separated. That is, the production decision does not depend on the utility function or the probability distribution of random variables, and the separation theorem holds even though basis risk exists. The firm fully hedge in currency market, but does not fully hedge in commodity market. When we assume unbiasedness in commodity futures market, i) if $E\tilde{p} > (Rp_f - K)$, then a short position is taken in futures and a short spot position is taken in the domestic market, ii) if $E\tilde{p} = (Rp_f - K)$, the firm does not use the futures market, iii) if $E\tilde{p} < (Rp_f - K)$, a long position is taken in futures and a long spot position is taken in the domestic market.

Second, when $Q_f \geq 0$ and Q_d is unrestricted if $Q_f = 0$, separation between the production and marketing decisions does not hold and firm's production decision is affected by the probability of random variables and risk attitudes. The optimal hedge ratio $(Z/Q) = \text{cov}(p, f) / \text{var}(f)$ which

is not full hedge.

Third, when Q_f is unrestricted and $Q_d \geq 0$ if $Q_d = 0$, the separation theorem holds only under specific conditions: $E\tilde{p} \leq (Rp_f - F)$, and the firm fully hedges in foreign exchange market.

Finally, when $Q_f \geq 0$ and $Q_d \geq 0$, if $Q_f = Q_d = 0$, the firm participates in the futures market as a speculator.

Because there are differences in the competitive firm's behavior between when the firm exports directly and exports by forward contract under uncertainty, and the volume of activity relative to that in futures markets, the model in this chapter is important. In next chapter, we'll examine the behavior of the importing offshore firm when the firm has access to both U.S. commodity futures market and the currency forward markets of its own country.

CHAPTER 5. BEHAVIOR OF THE IMPORTING OFFSHORE FIRM

5.1. Introduction

The international use of U.S. futures markets as a risk management tool becomes more important as international trade increases. Often, an offshore firm must use the U.S. futures market in order to hedge commodity price risk in international trading because futures markets only exist in U.S.. However, because futures contracts in the U.S. are traded in terms of U.S. dollars, the offshore firm faces different risks from the U.S. firm. That is, the offshore firm faces an exchange rate risk in that currency values can change between the time a futures contract is placed and the time a hedge is lifted. So movements in the exchange rate can affect both the level and variability of returns from commodity trading and futures transactions.

In this chapter, a model is developed to deal with the importing offshore firm from a small country. When the firm imports some materials as inputs for the production of final goods, it faces input price and exchange rate uncertainties. Therefore, the firm may want to hedge these uncertainties using futures contracts in U.S.. However, because this country is small, its currency is not traded in U.S. currency futures markets. But this small country has a well developed currency

forward market. So the firm can hedge its risks using the U.S. commodity futures market and its own currency forward market. The firm additionally faces basis risk in using U.S. commodity futures markets. This basis risk is important in using the commodity futures market internationally because traded commodities are not deliverable against contracts specified on futures exchanges, and the delivery date of the futures contract may not coincide with the import date of the product. In this case, the firm faces three uncertainties: exchange rate (\tilde{r}), commodity price (\tilde{p}) and futures price (\tilde{f}). We assume that the domestic currency price of final good is known with certainty or is relatively certain compared to input prices because our concern is the hedging behavior of the importing firm against input price and exchange rate uncertainties. The offshore firm pay transportation costs which are made in own country's currency.

5.2. The model

When the firm has access to both the U.S. commodity futures market and the currency forward markets of its own country, the firm's profits in domestic currency units can be expressed as

$$\tilde{\pi} = d G(L, M) - w L - \tilde{r}\tilde{p} M - kM + \tilde{r}(F - \tilde{f}) Z + (R - \tilde{r}) X$$

where L and M are the quantities of the two inputs in the production of the final commodity, M is the imported input, d is the unit price of final commodity, k is the unit transportation cost of import, $G(L, M)$ is a production function which satisfies the properties (i.e. $G_L > 0$, $G_M > 0$, $G_{LL} \leq 0$, $G_{MM} \leq 0$ and $(G_{LL}G_{MM} - G_{LM}^2) \geq 0$), w is the unit price of input L , \tilde{p} is the foreign price of the input in foreign currency in period $(t+1)$, \tilde{f} is the commodity futures price at the maturity date, and other variables are the same as before.

The optimization problem is

$$\begin{aligned} \text{Max}_{M, L, Z, X} \quad & EU(\tilde{\pi}) \quad \text{s.t} \\ \tilde{\pi} = & d G(L, M) - w L - \tilde{r}\tilde{p} M - kM + \tilde{r}(F - \tilde{f}) Z + (R - \tilde{r}) X \end{aligned}$$

The first order conditions for an optimum are

$$E[U'(\pi)(dG_L - w)] = 0 \quad (5.1)$$

$$E[U'(\pi)(dG_M - \tilde{r}\tilde{p} - k)] = 0 \quad (5.2)$$

$$E[U'(\pi)\tilde{r}(F - \tilde{f})] = 0 \quad (5.3)$$

$$E[U'(\pi)(R - \tilde{r})] = 0 \quad (5.4)$$

Rearranging equations (5.1) and (5.2) gives:

$$G_L = w/d \quad (5.5)$$

$$G_M = (\tilde{E}\tilde{r}\tilde{p} + k) / d + \text{cov}(U'(\pi), \tilde{r}\tilde{p}) / dEU'(\pi) \quad (5.6)$$

Equations (5.5) and (5.6) represent the marginal product of

inputs L and M respectively. Comparing the two equations (5.5) and (5.6) and applying Stein's theorem gives the following result:

$$\frac{G_M}{G_L} = \frac{(E\tilde{P}+k)}{w} - \frac{\lambda \text{cov}(\pi, rp)}{w} \quad (5.7)$$

where $\lambda = -EU''(\pi)/EU'(\pi)$ indicates the Arrow-Pratt measure of absolute risk aversion.

Equation (5.7) represents the ratio of marginal productivities between the two inputs. If there was no uncertainty, equation (5.7) would be $G_M/G_L = (rp+k)/w$, and the demand for the two inputs L and M would be determined by the non-stochastic prices (d, w, rp, k) . However, the demand for the two inputs is affected additionally by the stochastic factors due to uncertainty. In the absence of basis risk equations (5.5) and (5.6) are changed to:

$$dG_L = w$$

$$dG_M = RF+k$$

Then the demand for the two inputs L and M is determined by the non-stochastic price ratio w/d and $(RF+k)/d$. Thus there is a complete separation between the production and hedging decisions. However, due to basis risk, the separation does not hold.

Rearranging equations (5.2) and (5.3) and (5.4) and substituting equation (5.4) into (5.3) gives the following equations:

$$E[U'(\pi) dG_M] - E[U'(\pi) (\tilde{r}\tilde{p} + k)] = 0$$

$$E[U'(\pi) RF] - E[U'(\pi) (\tilde{r}\tilde{f})] = 0$$

$$E[U'(\pi) R] - E[U'(\pi) \tilde{r}] = 0$$

Applying Stein's theorem to above equations yields the following equations:

$$[dG_M - E\tilde{r}\tilde{p} - k] = \lambda [M \text{var}(rp) - F Z \text{cov}(rp, r) + Z \text{cov}(rp, rf) + X \text{cov}(rp, r)] \quad (5.8)$$

$$[RF - E\tilde{r}\tilde{f}] = \lambda [M \text{cov}(rp, rf) - F Z \text{cov}(rf, r) + Z \text{var}(rf) + X \text{cov}(rf, r)] \quad (5.9)$$

$$[R - E\tilde{r}] = \lambda [M \text{cov}(rp, r) - F Z \text{var}(r) + Z \text{cov}(rf, r) + X \text{var}(r)] \quad (5.10)$$

If we simultaneously solve equations (5.8), (5.9) and (5.10) and make the unbiasedness assumptions ($R=E\tilde{r}$ and $RF=E\tilde{r}\tilde{f}$), we get the following optimal decision values:

$$M^* = \frac{[dG_M - E\tilde{r}\tilde{p} - k] [\text{var}(rf) \text{var}(r) - \text{cov}^2(rf, r)]}{\lambda H} \quad (5.11)$$

$$Z^* = \frac{[dG_M - \tilde{E}r\tilde{p} - k] [\text{cov}(rp, r) \text{cov}(rf, r) - \text{cov}(rp, rf) \text{var}(r)]}{\lambda H} \quad (5.12)$$

$$X^* = \frac{[dG_M - \tilde{E}r\tilde{p} - k]}{\lambda H} * \\ [\text{cov}(rp, rf) (\text{cov}(rf, r) - F\text{var}(r)) - \text{cov}(rp, r) (\text{var}(rf) - F\text{cov}(rf, r))] \quad (5.13)$$

where $H \equiv \text{var}(rp)\text{var}(rf)\text{var}(r) [1 - \rho_{rpf}^2 - \rho_{rpr}^2 - \rho_{rfr}^2 + 2\rho_{rprf}\rho_{rpr}\rho_{rfr}]$.

Equation (5.11) represents the optimal import quantity.

Equations (5.12) and (5.13) represent the optimal commodity and currency hedge for imports. Because the firm hedges its price risk and exchange risk under the assumption of importing the input, $dG_M > (\tilde{E}r\tilde{p} + k)$ is a necessary condition for import and hedging without considering speculation.

If we assume that $H > 0$ (which must be tested empirically)

$M^* \geq 0$ iff $dG'(M) \geq (\tilde{E}r\tilde{p} + k)$ because $\text{var}(rf)\text{var}(r) > \text{cov}^2(rf, r)$ by the Cauchy-Schwartz inequality (Antonovitz and Nelson). Furthermore

$Z^* < 0$ iff $\rho_{rpr} \rho_{rfr} < \rho_{rprf}$ and $dG'(M) > \tilde{E}r\tilde{p}$

$X^* < 0$ iff $\text{cov}(rp, rf) [\text{cov}(rf, r) - F\text{var}(r)] < \text{cov}(rp, r) [\text{var}(rf) - F\text{cov}(rf, r)]$ and $G'(M) > (\tilde{E}r\tilde{p} + k)$.

From equations (5.11), (5.12) and (5.13) we can get the optimal hedge ratios.

$$\frac{Z^*}{M^*} = - \frac{\sqrt{\text{var}}(rp) [\rho_{rpf} - \rho_{rpr} \rho_{rfr}]}{\sqrt{\text{var}}(rf) [1 - \rho_{rfr}^2]}$$

or

$$\frac{Z^*}{M^*} = - \frac{[\text{var}(r) \text{cov}(rp, rf) - \text{cov}(rp, r) \text{cov}(rf, r)]}{[\text{var}(rf) \text{var}(r) - \text{cov}^2(rf, r)]} \quad (5.14)$$

$$X^* = FZ^* + M^* \frac{\sqrt{\text{var}}(rp)}{\sqrt{\text{var}}(r)} \frac{[\rho_{rpf} \rho_{rfr} - \rho_{rpr}]}{[1 - \rho_{rfr}^2]} \quad (5.15)$$

In equation (5.14) the firm considers exchange rate uncertainty in choosing the optimal commodity hedge ratio because exchange rate uncertainty affects the commodity hedge. That is, since basis risk and exchange rate risk is reflected, the optimal hedge ratio is not -1. In equation (5.15) the firm will set its optimal foreign exchange rate hedge equal to its commodity hedge in foreign currency (FZ^*) plus a factor to account for the unhedged risk of foreign sales due to a basis risk (the second term on the right-hand side of equation (5.15)).

Suppose that there is no basis risk (i.e., $\tilde{f} = \tilde{p}$). In this case, if the forward foreign exchange market is unbiased, namely, $E\tilde{r} = R$ and $E\tilde{r}\tilde{p} = RF$, then the optimal forward-futures contract becomes a full double-hedge, i.e. $Z^* = -M^*$ and $X^* = FZ^*$. However, because there is basis risk, the full double-hedge does not hold and the separation theorem

does not hold either.

5.3. The effects of introducing futures markets

In this section we examine the effect of introducing a futures market to the offshore country. Denote the optimal solutions with only the existence of the exchange rate forward market by π^e and in the absence of the commodity futures market and the currency forward market by π^n , as opposed to π^h , the optimal solution when both hedging markets exist.

Consider the case when the offshore firm has access to neither a commodity futures market nor an exchange rate forward market. The optimization problem is

$$\text{Max}_M EU(\pi^n) \quad \text{s.t.} \quad \pi^n = dG(L, M) - wL - \tilde{r}\tilde{p}M - kM$$

The first order conditions are

$$E[U'(\pi^n)(dG_L - w)] = 0$$

$$E[U'(\pi^n)(dG_M - \tilde{r}\tilde{p} - k)] = 0$$

If we solve above equations, we obtain the optimal quantity of import:

$$M^n = \frac{[dG_M - E\tilde{r}\tilde{p} - k]}{\lambda \text{var}(rp)} \quad (5.16)$$

Consider the case that there is only an exchange rate forward market. The optimization problem is

$$\text{Max}_{L, M, X} EU(\pi^e) \text{ s.t } \pi^e = dG(L, M) - wL - \tilde{r}\tilde{p}M - kM + (R - \tilde{r}) X$$

The first order conditions are

$$E[U'(\pi)(dG_L - w)] = 0$$

$$E[U'(\pi)(dG_M - \tilde{r}\tilde{p} - k)] = 0$$

$$E[U'(\pi)(R - \tilde{r})] = 0$$

If we simultaneously solve the first order conditions under the unbiased assumption ($R = E\tilde{r}$), then we get the following optimal solutions:

$$M^e = \frac{[dG_M - E\tilde{r}\tilde{p} - k]}{\lambda \text{var}(rp) (1 - \rho_{rpr}^2)} \quad (5.17)$$

$$X^e = - \frac{[dG_M - E\tilde{r}\tilde{p} - k] \text{cov}(rp, r)}{\lambda (1 - \rho_{rpr}^2) \text{var}(rp) \text{var}(r)} \quad (5.18)$$

M^e and X^e in equations (5.17) and (5.18) represent the optimal quantity of import and optimal currency hedge under the existence of only an exchange rate forward market.

Now examine the effect of introducing a futures market and a forward market to the offshore country. The method to check the effects of introducing the futures market is to compare $M^n(5.16)$, $M^e(5.17)$ and $M^h(5.11)$ when both hedging

markets exist.

First, compare equation (5.16) with (5.17) to check the effect of introducing the currency forward market.

$$\frac{M^e}{M^n} = \frac{1}{1 - \rho_{rpr}^2} > 1$$

Because the denominator $(1 - \rho_{rpr}^2) < 1$, M^e/M^n is greater than 1. That means, the optimal quantity of import is greater under the existence of only a currency forward market than without any hedging instruments. Therefore, if an exchange rate forward market is available, the offshore firm can increase its production by increasing imports since the production function $G(L, M)$ is a concave function and the exchange rate uncertainty is partially hedged.

Second, compare equation (5.16) with (5.11) to check the effect of introducing two hedging markets.

$$\frac{M^h}{M^n} = \frac{[1 - \rho_{rfr}^2]}{[1 - \rho_{rprf}^2 - \rho_{rpr}^2 - \rho_{rfr}^2 + 2\rho_{rprf}\rho_{rpr}\rho_{rfr}]} > 1 \quad (5.19)$$

If (numerator - denominator) > 0 in equation (5.19), then $M^h/M^n > 1$ and $M^h > M^n$. Because (numerator-denominator) = $\rho_{rprf}^2 - 2\rho_{rprf}\rho_{rpr}\rho_{rfr} + \rho_{rpr}^2$ and $(\rho_{rprf} - \rho_{rpr})^2 = \rho_{rprf}^2 - 2\rho_{rprf}\rho_{rpr} + \rho_{rpr}^2 > 0$ and $-1 \leq \rho_{rfr} \leq 1$, numerator $>$ denominator and $M^h > M^n$.

Therefore, if both hedging markets are available, the offshore firm can also increase its production by increasing imports

since commodity price and exchange rate uncertainties are hedged.

Lastly, compare equation (5.11) with (5.17) to analyze the difference between introducing both hedging markets and introducing only an exchange rate forward market.

$$\frac{M^h}{M^e} = \frac{[1 - \rho_{rpx}^2 - \rho_{rfx}^2 + \rho_{rpx}^2 \rho_{rfx}^2]}{[1 - \rho_{rpx}^2 - \rho_{rpx}^2 - \rho_{rfx}^2 + 2\rho_{rpx} \rho_{rpx} \rho_{rfx}]} > 1 \quad (5.20)$$

In equation (5.20) M^h is greater than M^e because (numerator - denominator) = $\rho_{rpx}^2 \rho_{rfx}^2 - 2\rho_{rpx} \rho_{rpx} \rho_{rfx} + \rho_{rpx}^2 = (\rho_{rpx} \rho_{rfx} - \rho_{rpx})^2 > 0$. So, the firm's production with two hedging markets is larger than the case with only a currency forward market.

Therefore, if the firm uses both hedging markets or only an exchange rate forward market, it increases production by increasing import stably with less uncertainties.

Next consider the effect of introducing a Korean futures market. When the firm has access to both the Korean commodity futures and currency forward markets, the firm's optimization problem can be expressed as:

$$\begin{aligned} \text{Max } EU(\tilde{\pi}) \quad \text{s.t} \\ \tilde{\pi} = dG(L, M) - wL - \tilde{r}_p M - kM + (F - \tilde{f})Z + (R - \tilde{r})X \quad (5.21) \end{aligned}$$

where F_k is the Korean futures price of the commodity in t for delivery in period $(t+1)$ and \tilde{q} is a Korean futures price of

the commodity at maturity date.

The first order conditions are

$$E[U'(\pi)(dG_L - w)] = 0 \quad (5.22)$$

$$E[U'(\pi)(dG_M - \tilde{r}\tilde{p} - k)] = 0 \quad (5.23)$$

$$E[U'(\pi)(F_k - \tilde{q})] = 0 \quad (5.24)$$

$$E[U'(\pi)(R - \tilde{r})] = 0 \quad (5.25)$$

If we simultaneously solve the first order conditions and assume unbiasedness, we can obtain the following optimal decision variables:

$$M^* = \frac{[dG_M - k - E\tilde{f}\tilde{p}]}{\lambda \text{var}(rp) [1 - \rho_{rpq}^2 - \rho_{rpr}^2]} \quad (5.26)$$

$$Z^* = \frac{-[dG_M - k - E\tilde{f}\tilde{p}] \text{cov}(rp, q)}{\lambda \text{var}(rp) \text{var}(p) [1 - \rho_{rpq}^2 - \rho_{rpr}^2]} \quad (5.27)$$

$$X^* = \frac{-[dG_M - k - E\tilde{f}\tilde{p}] \text{cov}(rp, r)}{\lambda \text{var}(rp) \text{var}(r) [1 - \rho_{rpq}^2 - \rho_{rpr}^2]} \quad (5.28)$$

The optimal hedge ratios are:

$$Z / M = - \text{cov}(rp, q) / \text{var}(f) \quad (5.29)$$

$$X = - M \text{cov}(rp, r) / \text{var}(r) = - M E\tilde{p} \quad (5.30)$$

The optimal commodity hedge and the optimal amount of import depend on the correlation between r_p and q . As correlation between r_p and q increases, imports and the optimal hedge ratio increase.

Now examine the effects of introducing a commodity futures and a currency forward markets directly into Korea. If we compare equation (5.16) with (5.26)

$$\frac{M^k}{M^n} = \frac{1}{[1 - \rho_{rpq}^2 - \rho_{rpr}^2]} > 1 \quad (5.31)$$

where M^k represents the optimal amount of import in the case of introducing a commodity futures and currency forward market directly into Korea. Because the denominator $(1 - \rho_{rpq}^2 - \rho_{rpr}^2) < 1$, M^k/M^n is greater than 1. That is, the optimal quantity of import is greater when introducing a commodity futures and currency forward market directly into Korea than under without both hedging instruments.

5.4. Summary

This chapter examined the behavior of an importing offshore firm which faces input price and exchange rate uncertainties when it has access to U.S. commodity futures markets and its own currency forward markets. We find that the demand for input M is determined by the stochastic and

nonstochastic prices and risk attitudes, and the separation between the production and hedging decisions does not hold due to basis risk. The firm considers exchange rate uncertainty in the optimal commodity hedge because exchange rate uncertainty affects the commodity hedge. The firm sets its optimal foreign exchange rate hedge equal to its commodity hedge in foreign currency plus a factor to account for the unhedged risk of foreign sales due to basis risk.

In examining the effect of introducing a futures market and a forward market to the offshore country, there are three results: First, introducing only an exchange rate forward market to the offshore country makes the offshore firm increase its production by increasing imports since the exchange rate uncertainty is partially hedged. Secondly, introducing both hedging markets to the offshore country also encourages the offshore firm to increase its production by increasing imports since commodity price and exchange rate uncertainties are hedged. Finally, the firm's production with two hedging markets is larger than the case with only a currency forward market. Also when commodity futures markets are directly made in Korea, the hedging role of the futures market is still effective.

This chapter is important because if the offshore firm uses U.S. commodity futures markets, it can increase its production and profits by increasing imports with less

uncertainties. This of course assumes that the firm is a small player in the domestic market and will not affect the price of the product. Thus this analysis is clearly partial equilibrium. This study also develops a hedging strategy appropriate for importers. In next chapter, this model will be used for empirical work.

CHAPTER 6. EMPIRICAL STUDY

6.1. Introduction

In this chapter we empirically consider the optimal hedging strategies of a Korean grain importing firm using the model developed in Chapter 5. Although the model of the importing offshore firm in Chapter 5 can be applied to many products including oil, metals and lumber, we choose agricultural products such as corn, soybean, and wheat which are used as inputs because they have severe price fluctuations relative to industrial products due to production uncertainty. The grain importing firm's concern is how to import grain with less price uncertainty because with small domestic production the international uncertainty in these prices severely affects the risk position of the firm.

In the empirical application, we consider a Korean corn importing firm which also feeds livestock. This firm could be a large integrated firm (i.e., Samsung), or a cooperative such as the National Livestock Cooperatives Federation or the Korean Feed Association. In either case the firm is assumed to purchase feed for use as an input in production. The cooperative is assumed to act as a vertically integrated firm. Suppose this firm commits itself in the current period to

producing livestock for the next period, and this production requires the imported feed as an input. The foreign currency price of the grain is random, so there is risk.

When the firm commits itself in current period to producing final goods for the next period, it can buy a three month maturity futures contract in order to hedge the input price risk. Since corn is the most important imported feed grain the analysis will focus on corn imports and futures trading.

Corn imports are controlled by the government in Korea by using an import quota. Since 1984, individual feed mills and the National Livestock Cooperatives Federation as well as the Korean Feed Association have been authorized to import feed corn. Korea's livestock economy is directly related to the import corn. The importing companies (for example, Samsung overseas funding company) have a union for grain importing. The union helps make the decision on the import quota. Once the quota is allocated, the individual firm tries to purchase corn as cheaply as possible. The firms market corn in Korea, or use it directly for livestock production. The union and the government does not set the margins of these importing companies for corn sales. According to the Korean Rural Economics Institutes and Department of Korean Government (Department of Agricultural, Forestry, and Fishery), it might be possible for these importing companies to collude each

other in setting prices. However, there are many importing companies and import quota is very flexible. So we assume the current market is perfect competition.

In Korea the NACF (National Agricultural Cooperative Federation), as the government buying agent, purchases all corn offered by farmers, selling the great bulk of it to feed mills and the rest to companies of the Korean Corn Processors Association. Feed manufacturers are forced to buy domestic corn from the NACF at its cost of acquisition and handling, which continues to be several times greater than the import price. Thus firms will purchase as much corn as possible on the international market and domestic production is not an important consideration.

Following Goodwin, Grennes and Wohlgenant, "international grain trade is highly organized and shipments may flow through several agents before reaching end-users. A large percentage (85%-90%) of U.S. grain exports are handled by five companies: Cargill, Continental, Bunge and Born, Louis Dreyfus, and Andre Garnac (Davies). These companies conduct marketing and arbitrage activities as they arrange the exchange of grain and oilseed commodities between domestic producers and foreign end-users." If a Korean importing firm must buy corn in the U.S., it may face oligopoly in import market. However, the Korean importing firm can also buy grain in Argentina, China, Thailand, South Africa, France, Canada and other countries.

Because of that the market is almost perfectly competitive. So, we will consider grain market as a competitive market.

The period studied is 1980-1990 and midmonth closing prices (f) from the Wall Street Journal are used for the nearby corn futures contract. The spot prices (p) are the monthly U.S. average export price obtained from Feed Situation and Outlook Report. The monthly average Korean Won/US dollar exchange rate (r) are obtained from the International Financial Statistics. There are seasonal fluctuations which occur within a year in many agricultural products because they are primarily related to seasonal factors, such as the weather. Many economists try to remove this seasonality.

Pierce (1980) argues that they do because

..... our ability to recognise, interpret, or react to important nonseasonal movements in a series (such as turning points and other cyclical events, emerging patterns, or unexpected occurrences for which possible cause are sought) is hindered by the presence of seasonal movements.

However, seasonality may be valuable information in commodity hedging strategy because seasonality gives regular variation in price series. So there is no strong reason to remove seasonality in grain product hedging strategy.

The purpose of this chapter is to estimate the optimal commodity and currency hedge ratios derived in Chapter 5 and the effects of introducing futures markets for a Korean grain

importing firm.

The optimal commodity hedge ratio is

$$\frac{Z^*}{M^*} = - \frac{\sqrt{\text{var}}(rp) [\rho_{rprf} - \rho_{rpr} \rho_{rfr}]}{\sqrt{\text{var}}(rf) [1 - \rho_{rfr}^2]} \quad (6.1)$$

where M^* = optimal amount of the commodity to be imported.

Z^* = optimal amount of the commodity sold (if $Z > 0$) or purchased (if $Z < 0$) in the commodity futures market.

The optimal currency hedge ratio is

$$\frac{X^*}{FM^*} = \frac{Z^*}{M^*} + \frac{\sqrt{\text{var}}(rp) [\rho_{rprf} \rho_{rfr} - \rho_{rpr}]}{F \sqrt{\text{var}}(r) [1 - \rho_{rfr}^2]} \quad (6.2)$$

where X^* = optimal amount of foreign exchange purchased (if $X < 0$) in the currency forward market.

The effects of introducing futures markets are

$$\frac{M^e}{M^n} = \frac{1}{1 - \rho_{rpr}^2} \quad (6.3)$$

$$\frac{M^h}{M^n} = \frac{[1 - \rho_{rfr}^2]}{[1 - \rho_{rprf}^2 - \rho_{rpr}^2 - \rho_{rfr}^2 + 2 \rho_{rprf} \rho_{rpr} \rho_{rfr}]} \quad (6.4)$$

$$\frac{M^h}{M^e} = \frac{[1 - \rho_{rpr}^2 - \rho_{rfr}^2 + \rho_{rpr}^2 \rho_{rfr}^2]}{[1 - \rho_{rprf}^2 - \rho_{rpr}^2 - \rho_{rfr}^2 + 2 \rho_{rprf} \rho_{rpr} \rho_{rfr}]} \quad (6.5)$$

where M^e = optimal amount of the commodity to be imported in

the only existence of the exchange rate forward market.

M^n = optimal amount of import in the absence of the commodity futures market and the currency forward market.

M^h = optimal amount of import when both hedging markets exist.

So, we need the variance, covariance and correlation coefficients from the data set obtained to estimate the above equations.

In theoretical model, when the decision maker solves the model, the solution of the decision variables will be a function of population parameters which are assumed to be known. However, in empirical applications, because these population parameters are actually unknown to the decision maker, these are simply replaced by the sample parameter estimates. This leads to an additional source of uncertainty, called estimation risk (Chalfant, Collender and Subramanian). So, because of the existence of this estimation risk, the estimation results in this empirical part must be interpreted with caution.

6.2. Unit root test

When the time series under investigation are stationary, they have a clear meaning. If the series under investigation are nonstationary, the usual distributional results and tests of significance are no longer valid. Engle and Granger (1987)

show that since nonstationary variables have infinite variances that make the F-tests or t-tests invalid, standard hypothesis testing does not apply to time series with unit roots. First of all, we need test whether a series is stationary. Dickey and Fuller (1979) have developed unit root tests which provide an easy method of testing whether a series is nonstationary. The rejection of the unit root hypothesis provides the necessary condition to conclude that a series is stationary, but not a sufficient condition. If a series X_t has a stationary, invertible ARMA representation after first differencing, it is said to be integrated of order 1, i.e., $X_t \sim I(1)$. A stationary series is denoted by an $I(0)$ series. $I(d)$ represents that a series needs to be differenced d times to become stationary. The order of integration can be inferred by testing for unit roots.

Let's consider a time series (X_t) which is difference stationary,

$$X_t = a + b X_{t-1} + e_t \quad (6.6)$$

If $|b| < 1$, then X_t is said to be stationary. If $b = 1$, X_t is difference-stationary. The usual t-statistic for testing the null hypothesis that b is equal to one is not valid here. Therefore, we can reparameterize equation (6.6) and (6.7) by subtracting X_{t-1} on both sides of equation (6.6).

$$\Delta X_t = a + (b-1) X_{t-1} + e_t \quad (6.7)$$

where $\Delta = (1-L)$, L is the lag operator.

One can estimate the equation (6.7) with OLS and compare the t-statistic on the coefficient of X_{t-1} with the critical value τ provided in Fuller (1976, p 373). This procedure is called Dickey-Fuller (DF) test and valid when e_t is a white noise process (serially uncorrelated and homoscedastic disturbance). This assumes that the first order autoregressive model is correct. A simple way to account for the serial correlation is to write equation (6.7) as

$$\Delta X_t = a + (b-1) X_{t-1} + \sum_{i=1}^n r_i \Delta X_{t-i} + e_t \quad (6.8)$$

and n is selected to be large enough to ensure that the residuals e_t are white noise. One can estimate equation (6.8) with OLS and compare the t-statistic on the coefficient of X_{t-1} with the same critical value provided in Fuller. This procedure is called the Augmented Dicky-Fuller (ADF) test. The ADF tests examine

$H_0 : b = 1$, the null hypothesis of unit root process against
 $H_1 : b < 1$. The rejection of H_0 implies that the series X_t is stationary.

Table 1 reports the Augmented Dicky-Fuller (ADF) tests for the stationarity of the exchange rate (r), futures price (f), spot price (rp) and futures price (rf) in terms of a

Korean Won. The null hypothesis that there is a unit root can be accepted at the 95% confidence level if the τ -statistic falls below 2.89 in absolute value for a sample of 100 observations. From table 1 it can be seen that all the price series follow a process with a unit root. In this paper, if a unit root was found then series was differenced and the differenced series was tested again for a second unit root. The results are given in table 2. It is apparent that the hypothesis that the first-order differences of the prices is $I(1)$ is rejected significantly and, therefore, all the prices are integrated of the first order; that is, $I(1)$.

Standard unit root tests routinely fail to reject the null hypothesis of a unit root for many economic time series. Because in empirical work the unit root is the null hypothesis to be tested, and the way in which classical hypothesis testing is carried out, the standard procedure ensures that the null hypothesis is accepted unless there is strong evidence against it (Kwiatkowski, Phillips, Schmidt and Shin (KPSS)). Therefore, KPSS suggest that, in trying to decide by classical methods whether economic data are stationary or integrated, it would be useful to perform tests of the null hypothesis of stationary as well as tests of the null hypothesis of a unit root. They provide a test of the null hypothesis of stationary against the alternative of a unit root. Let's introduce the

Table 1. Test for unit roots in f, r, rp and rf

Pseudo t-statistics	
ADF specification (T = 129)	
f	-2.0179
r	-2.1183
rp	-2.2281
rf	-2.3522

Table 2. I(1) test

Pseudo t-statistics	
ADF specification (T = 126)	
f	-7.4415
r	-3.7081
rp	-6.695
rf	-7.4531

KPSS's method. Let y_t , $t=1,2,3,\dots,T$, be the observed series for which we wish to test stationary. They assume that they can decompose the series into the sum of a deterministic trend, a random walk, and a stationary error:

$$y_t = at + r_t + \epsilon_t \quad (6.9)$$

Here $r_t = r_{t-1} + u_t$, where, the u_t are iid($0, \sigma_u^2$). The initial value r_0 is treated as fixed and serves the role of an intercept. The stationarity hypothesis is simply $\sigma_u^2 = 0$. Since ϵ_t is assumed to be stationary, under the null hypothesis y_t is stationary around a level (r_0) rather than around a trend.

Let e_t , $t=1,2,3,\dots,T$, be the residuals from the regression of y on an intercept and time trend. Let $\hat{\sigma}_\epsilon^2$ be the estimate of the error variance from this regression (the sum of squared residuals, divided by T). Define the partial sum process of the residuals:

$$S_t = \sum_{i=1}^t e_i, t=1,2,\dots,T.$$

Then the LM (Lagrangean multiplier) statistic is

$$LM = \sum_{t=1}^T S_t^2 / \hat{\sigma}_\epsilon^2 \quad (6.10)$$

Furthermore, in the event of the null hypothesis of level

stationary instead of trend stationarity, e_t is the residual from the regression of y on intercept only. The test is an upper tail test. However, the series to which the stationary test will be applied are typically highly dependent over time, and so the iid error assumption under the null is unrealistic. To allow for quite general forms of temporal dependence, they define the long-run variance as

$$\sigma^2 = \lim_{T \rightarrow \infty} T^{-1} E(S_T^2)$$

which will enter into the asymptotic distribution of the test statistic. A consistent estimator of σ^2 , say $s^2(l)$, can be constructed from the residuals e_t ,

$$s^2(l) = T^{-1} \sum_{t=1}^T e_t^2 + 2T^{-1} \sum_{s=1}^l w(s, l) \sum_{t=s+1}^T e_t e_{t-s}$$

Here, $w(s, l)$ is an optimal weighting function that corresponds to the choice of a special window. They use the Bartlett window $w(s, l) = 1 - s/(l+1)$. For consistency of $s^2(l)$, it is necessary that the lag truncation parameter $l \rightarrow \infty$ as $T \rightarrow \infty$. So, when the errors are not iid, the appropriate denominator of the test statistic is an estimate of σ^2 instead of σ_e^2 . The numerator of the test statistic is

$$\eta = T^{-2} \sum S_t^2$$

Then the test statistic is

$$\hat{\eta}_{\mu} = T^{-2} \sum S_t^2 / s^2(1)$$

$$\hat{\eta}_{\tau} = T^{-2} \sum S_t^2 / s^2(1)$$

where μ represents the level stationary and τ represents the trend stationary.

Table 3 represents the upper tail critical values for level and trend stationarity. Now we apply KPSS's method for the stationarity test to the data of f , r , rf , rp . In table 4, we present the η_{μ} test statistic for the null hypothesis of stationarity around a level, and the η_{τ} test statistic for the null hypothesis of stationarity around a deterministic linear trend. We can reject the null hypothesis of level or trend stationary at usual critical levels for all series. This means that all series may be nonstationary. Since this KPSS test is intended to complement unit root tests, such as the Dickey-Fuller tests, by testing the null hypothesis of both the unit root (Dickey-Fuller method) and the stationarity (KPSS method), use of both helps confirm whether the series is stationary or not. Because in ADF test, all series fail to reject the null hypothesis of unit root, and in KPSS test, all series reject the null hypothesis of stationary, we can say that all series are non-stationary.

Table 3. Upper tail critical values for η_μ and η_τ

η_μ : Upper tail percentiles of the level stationarity				
Critical level:	0.10	0.05	0.025	0.01
Critical value:	0.347	0.463	0.574	0.739
η_τ : Upper tail percentiles of the trend stationarity				
Critical level:	0.10	0.05	0.025	0.01
Critical value:	0.119	0.146	0.176	0.216

Table 4. η_μ and η_τ tests for stationarity applied to
f, r, rp, rf.

Series	Lag truncation parameter (l)			
	1	2	3	4
For level stationarity (η_μ)				
f	1.59120	0.98346	0.72470	0.58256
r	1.20836	0.73444	0.53180	0.41970
rp	0.94258	0.58604	0.43500	0.35238
rf	1.18340	0.73892	0.55018	0.44666
For trend stationarity (η_τ)				
f	0.29635	0.18575	0.13923	0.11424
r	1.01543	0.61625	0.44559	0.35119
rp	0.41708	0.26132	0.19570	0.16018
rf	0.32433	0.20519	0.15516	0.12827

6.3. Estimation

Because all series have a unit root, the problem is to obtain the correct variance and covariance. If we difference the series, the differenced series are stationary, but those series can not appropriately represent the original series because the unconditional variances are different. The way to solve that problem is to use the conditional mean, variance and covariance. In Myers and Thompson (1989), they consider conditional moments that depend on information available at the time the hedging decision is made when they get the optimal hedge ratio. They assume that spot and futures prices for a commodity are generated by the following linear equation:

$$p_t = I_{t-1} \alpha + u_t$$

$$f_t = I_{t-1} \beta + v_t$$

where I_{t-1} is a vector of variables known at $t-1$ that help predict p_t and f_t . Examples of variables that might appear in I_{t-1} are lagged values of spot and futures prices, production, storage, exports, and consumer income all dated $t-1$ and earlier. By applying the conditional (on I_{t-1}) covariance operator to above equation, it is found that

$$\sigma_p^2 = \text{var}(u_t | I_{t-1})$$

$$\sigma_f^2 = \text{var}(v_t | I_{t-1}), \text{ and}$$

$$\sigma_{pf} = \text{cov}(u_t, v_t | I_{t-1}).$$

The problem has been reduced to one of obtaining an estimate of Φ .

$$\Phi = \frac{1}{T} \begin{vmatrix} u'u & u'v \\ v'u & v'v \end{vmatrix}$$

$$\text{Optimal hedge ratio} = \frac{v'u}{v'v}$$

However, their method of obtaining optimal hedge ratio may not be applicable in international market because optimal hedge ratio is not a simple ratio between covariance and variance of futures and spot price, but a series is multiplied by an exchange rate.

By Fama's definition, a "market is efficient" if new information is fully and timely reflected in price. That is, market prices adjust to new information rapidly and completely and that these adjustments are, on average, correct so that the adjusted prices are market clearing. If we follow the market efficiency hypothesis, information conditional on its own past value may be the most valuable and the price series itself may include all relevant information revealed. So we use the AR(1) model in order to obtain variance and covariance

of the price series under the conditional information set.

Let X_t be the related time series which is nonstationary. X_t can be represented by the Autoregressive process (AR(1)).

$$X_t = a + \beta X_{t-1} + e_t \quad (6.11)$$

Since X_t is nonstationary, the variance of X_t is infinite. Also we can not estimate β correctly because X_t and X_{t-1} are nonstationary. So we need to use a first difference.

$$X_t - X_{t-1} = \beta (X_{t-1} - X_{t-2}) + (e_t - e_{t-1}) \quad (6.12)$$

Since $(X_t - X_{t-1})$ and $(X_{t-1} - X_{t-2})$ are stationary, we can correctly estimate β . However, $\text{var}(X_t - X_{t-1})$ is not the same as $\text{var}(X_t)$ because $\text{var}(X_t - X_{t-1}) = \text{var}(X_t) + \text{var}(X_{t-1}) - 2\text{cov}(X_t, X_{t-1}) \neq \text{var}(X_t)$. Because of that we can not use the unconditional variance of the differenced series. Instead of that we use the conditional variance of the differenced series because $\text{var}_{t-1}(X_t - X_{t-1}) = \text{var}_{t-1}(X_t)$. Here var_{t-1} represents the conditional variance which depends upon the information available at time $t-1$. Rearranging equation (6.12)

$$X_t = X_{t-1} + \beta (X_{t-1} - X_{t-2}) + e_t - e_{t-1} \quad (6.13)$$

If we take a conditional expectation of equation (6.13),

$$E_{t-1} X_t = X_{t-1} + \beta (X_{t-1} - X_{t-2}) - e_{t-1} \quad (6.14)$$

where E_{t-1} is the conditional expectation which depends on information available at time $t-1$.

To get conditional variance we use

$$X_t - E_{t-1} X_t = e_t \quad (6.15)$$

Here, $\text{var}_{t-1}(e_t)$ is finite and represents the conditional variance of the original series. If we consider another two series Y_t and Z_t , then we can get following by using the same procedure:

$$Y_t - E_{t-1} Y_t = n_t \quad (6.16)$$

$$Z_t - E_{t-1} Z_t = v_t \quad (6.17)$$

From equation (6.15), (6.16) and (6.17) we can get the conditional variance and covariance matrix (Ω) of the shock e_t and n_t .

$$\Omega = \frac{1}{T} \begin{vmatrix} e'e & e'n & e'v \\ n'e & n'n & n'v \\ v'e & v'n & v'v \end{vmatrix} \quad (6.18)$$

where e , n and v are vectors of residuals from estimating equation (6.15), (6.16) and (6.17), respectively, and T is the

number of observation. The conditional mean and variance for f , r , rf and rp are reported in Table 5. The means and standard deviations of the unanticipated changes in f , r , rf and rp are presented in Table 6. Estimating equation (6.18) gives Tables 7 and 8 which present the covariance and correlations among the unanticipated changes for f , r , rf and rp . If we apply values in Table 5, 6, 7 and 8 to equations (6.1), (6.2), (6.3), (6.4) and (6.5), we can following results:

$$\frac{Z^*}{M^*} = -\frac{\sqrt{\text{var}}(rp) [\rho_{rprf} - \rho_{rpr} \rho_{rfr}]}{\sqrt{\text{var}}(rf) [1 - \rho_{rfr}^2]} = -0.67232 \quad (6.1')$$

$$\frac{X^*}{FM^*} = \frac{Z^*}{M^*} + \frac{\sqrt{\text{var}}(rp) [\rho_{rprf} \rho_{rfr} - \rho_{rpr}]}{F \sqrt{\text{var}}(r) [1 - \rho_{rfr}^2]} = -0.61 \quad (6.2')$$

$$\frac{M^o}{M^n} = \frac{1}{1 - \rho_{rpr}^2} = 1.012 \quad (6.3')$$

$$\frac{M^h}{M^n} = \frac{[1 - \rho_{rfr}^2]}{[1 - \rho_{rprf}^2 - \rho_{rpr}^2 - \rho_{rfr}^2 + 2 \rho_{rprf} \rho_{rpr} \rho_{rfr}]} = 2.848 \quad (6.4')$$

$$\frac{M^h}{M^o} = \frac{[1 - \rho_{rpr}^2 - \rho_{rfr}^2 + \rho_{rpr}^2 \rho_{rfr}^2]}{[1 - \rho_{rprf}^2 - \rho_{rpr}^2 - \rho_{rfr}^2 + 2 \rho_{rprf} \rho_{rpr} \rho_{rfr}]} = 2.8148 \quad (6.5')$$

In order to get the optimal currency hedge ratio in equation (6.2'), we need to assume that the commodity futures markets are unbiased ($E_{t-1} f_t = F$). However, since f_t series is nonstationary, we must get a conditional mean ($E_{t-1} f_t = F = 2.6646$).

Equation (6.1') represents the optimal commodity hedge ratio (-0.67) which is an underhedge because basis risk and exchange rate risk is reflected. Equation (6.2') is the optimal currency hedge ratio (-0.61) which is not a perfect hedge. So when the firm imports corn in the U.S. commodity market, it also buys 67.2% of corn in the U.S. corn futures market and also buys 61% of currency (U.S. \$) among its value (FM^*) in the currency forward market. Equation (6.1') shows that when a Korean importing firm uses U.S. commodity futures market, as conditional correlation between r_p and r_f is higher, the optimal commodity hedge ratio increases because the correlation between r_p and r_f is positive.

In equation (6.3') the optimal quantity of import under the existence of only a currency forward market is 1.012 times greater than under without any hedging instruments. Equation (6.4') shows that the import can be increased 2.848 times when the Korean firm uses both hedging markets. Equation (6.5') shows that the optimal amount of import under the existence of both hedging instruments is 2.81 times greater than under the existence of only a currency forward hedging instrument. That

Table 5. Conditional mean and variance for f , r , rf and rp

	f	r	rf	rp
Mean	2.6646	757.31	1996.7	2157.6
Variance	0.064	18.025	19237	13373

Table 6. Summary of means and other statistics of the unanticipated changes in f , r , rf and rp

	ef	er	erf	erp
Mean	-0.0015674	0.25676	0.03025	0.37329
Variance	0.064	18.025	19237	13373
Minimum	-0.62122	-12.765	-485.27	-448.18
Maximum	1.2176	20.07	644.57	542.06

where $er = r - E_{t-1}r =$ unanticipated changes in r

$erf = rf - E_{t-1}rf =$ unanticipated changes in rf

$erp = rp - E_{t-1}rp =$ unanticipated changes in rp

$ef = f - E_{t-1}f =$ unanticipated changes in f

Table 7. Correlation matrix of variables

er	1.000			
erf	0.14136	1.000		
erp	0.10788	0.80551	1.000	
ef	0.02657	0.68953	0.58399	1.000
	er	erf	erp	ef

Table 8. Covariance matrix of variables

er	18.025			
erf	83.235	19237		
erp	52.966	12920	13373	
ef	0.02854	24.194	17.085	0.064
	er	erf	erp	ef

where $er = r - E_{t-1}r =$ unanticipated changes in r

$erf = rf - E_{t-1}rf =$ unanticipated changes in rf

$erp = rp - E_{t-1}rp =$ unanticipated changes in rp

$ef = f - E_{t-1}f =$ unanticipated changes in f

means, the introduction of the currency forward market without introducing the commodity futures market has little hedging effect in commodity trading (import) in Korea. If a Korean grain importing firm uses the U.S. futures markets, it can reduce inefficiencies caused by unstable prices and increase the amount of import, and on the other hand the U.S. firm can increase its export. So, two country's trade may be enhanced. Alternatively the Korean government could help domestic traders by entering into futures contracts in Chicago on behalf of domestic traders. However, given the size of the firms included, this is probably not warranted.

CHAPTER 7. SUMMARY AND CONCLUSIONS

Increased trading activities and expanding markets in recent years and related economic or political shocks have created higher and more volatile prices resulting in greater uncertainty in decision making. Since the break down of the fixed exchange rate regime in the early 1970s, foreign exchange rates have fluctuated wildly. So exchange rate risk has become an important issue for management of firms with international transactions. This environment has led to an increased interest in the use of futures markets, and commodity and currency futures markets have rapidly developed and been actively utilized in recent years as a way of efficiently coping with exchange rate and price risks.

The purpose of this dissertation is to analyze the competitive firm's behavior under the price and exchange rate uncertainty when the firm participates in futures markets to hedge against these risks. This dissertation deals with three theoretical models which consider three different firms, and one empirical study. In two of the theoretical models, four possible cases are considered according to the trade restrictions.

In the first model, when Q_f and Q_o are unrestricted, the production decision does not depend on the risk attitudes or

probability distribution of the random variables. That is, the separation theorem holds. However, marketing decisions (and the hedging decision) are affected by uncertainty even though there are hedging instruments. Whether the firm exports or imports depends on the relative prices between the expected net foreign price (ENFP) and the certain futures price (F):

i) If $ENFP \gg F$, the firm exports and optimal hedge $Z = -Q_0 + Q_f \text{cov}(rs,p)/\text{var}(p)$, ii) If $ENFP > F$ (not very large), $Z = Q_0 + Q_f \text{cov}(rs,p)/\text{var}(p)$, iii) If $ENFP < F$, the firm imports and $Z = Q_0 - Q_f \text{cov}(rs,p)/\text{var}(p)$. When the commodity or currency futures price is biased, the firm underhedges under normal backwardation and overhedges under contango.

In restricted model, if $Q_f = 0$, the separation theorem still holds, but if $Q_0 = 0$, separation does not hold because firm 's production is based on the foreign market conditions with uncertainty. If $Q_0 = Q_f = 0$, the firm does not produce and participate in the futures market as a speculator.

When futures markets are newly introduced to the market, production increases since the producer (or hedger) can make a production decision with certainty by using futures markets. If basis risk in commodity futures market is introduced to the complete market model, the production and hedging decision are not separated, and the production and marketing (and hedging) decisions are affected by uncertainty.

In the second theoretical model, the firm can export or

import forward and supply or purchase output in the domestic market under price and exchange rate uncertainties and basis risk. In the unrestricted case, the production and hedging decisions are separated even though basis risk exists. The firm fully hedges in the currency market, but does not fully hedge in commodity market due to basis risk. If $E\tilde{p} > (Rp_f - k)$, then a short position is taken in futures and a short spot position is taken in the domestic market. If $E\tilde{p} = (Rp_f - k)$, the firm does not use the futures market. If $E\tilde{p} < (Rp_f - k)$, a long position is taken in futures and a long spot position is taken in the domestic market. In a restricted case, if $Q_f = 0$, the separation theorem does not hold, the production and marketing (and hedging) decision is affected by uncertainty, and the optimal hedge ratio $(Z/Q) = \text{cov}(p, f) / \text{var}(f)$, which is not a full hedge. If $Q_d = 0$, the separation theorem holds only under specific conditions: $E\tilde{p} \leq (Rp_f - F)$, and the firm fully hedges in the foreign exchange market.

In the third theoretical model in Chapter 5, the importing offshore firm faces input price and exchange rate uncertainties when it has access to U.S. commodity futures markets and its own currency forward markets. The separation between the production and hedging decision does not hold due to basis risk, and the demand for input M is determined by the stochastic and nonstochastic prices and risk attitudes. The firm considers exchange rate uncertainty in the optimal

commodity hedge because exchange rate uncertainty affects the commodity hedge. The firm set its optimal foreign exchange rate hedge equal to its commodity hedge in the foreign currency plus a factor to account for the unhedged risk of foreign sales due to a basis risk. Introducing both hedging markets to the offshore country makes the offshore firm increase its production by increasing imports since the production function is concave, and commodity price and exchange rate uncertainties are hedged.

In the empirical study, the optimal commodity and currency hedge ratios and the effect of introducing futures markets are estimated for a Korean grain importing firm. In the estimation problem, the time series are investigated as to whether they are stationary or not using the Dickey-Fuller (DF) method which tests the null hypothesis of unit root, and KPSS method which tests the null hypothesis of stationarity. Because in the DF test, all series fail to reject the null hypothesis of unit root and in KPSS test all series reject to the null hypothesis of stationarity, we can say that all series are non-stationary. So we use the conditional moments method in order to correctly obtain more information and to solve the non-stationary problem.

The results show that optimal commodity and currency hedges are underhedges because risk is not completely hedged. When a Korean importing firm uses the U.S. commodity futures

markets, as conditional correlation between r_p and r_f is higher, the optimal commodity hedge ratio increases. When the Korean importing firm uses both hedging markets, the import can be increased 2.848 times. The introduction of the currency forward market without introducing the commodity futures market is not very effective for commodity trading hedges in Korea. If a Korean firm uses the U.S. futures markets, the Korean firm can reduce inefficiencies caused by unstable prices and increase the amount of import, and on the other hand a U.S. firm can increase its export. Therefore, both two country's trade could be enhanced.

The first and second models provide some insight into the risk shifting role of futures markets and marketing strategy when the both hedging markets are available. The third theoretical model develops a hedging strategy appropriate for importers using offshore futures markets. The empirical study directly estimates optimal hedges.

Limitations of this dissertation must be recognized. If uncertainties in production (in Chapter 3 and 4) and domestic prices of final goods (in Chapter 5) are introduced, the optimality conditions should be modified and would be much more complicated. If estimation risk in empirical study is corrected, the empirical results will be more precise. Sometimes in the market environment an oligopolistic market model instead of a competitive market model may be reasonable

to explain real world. So the oligopolistic market model should be examined in future research.

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APPENDIX. SUPPLEMENTARY FIGURES

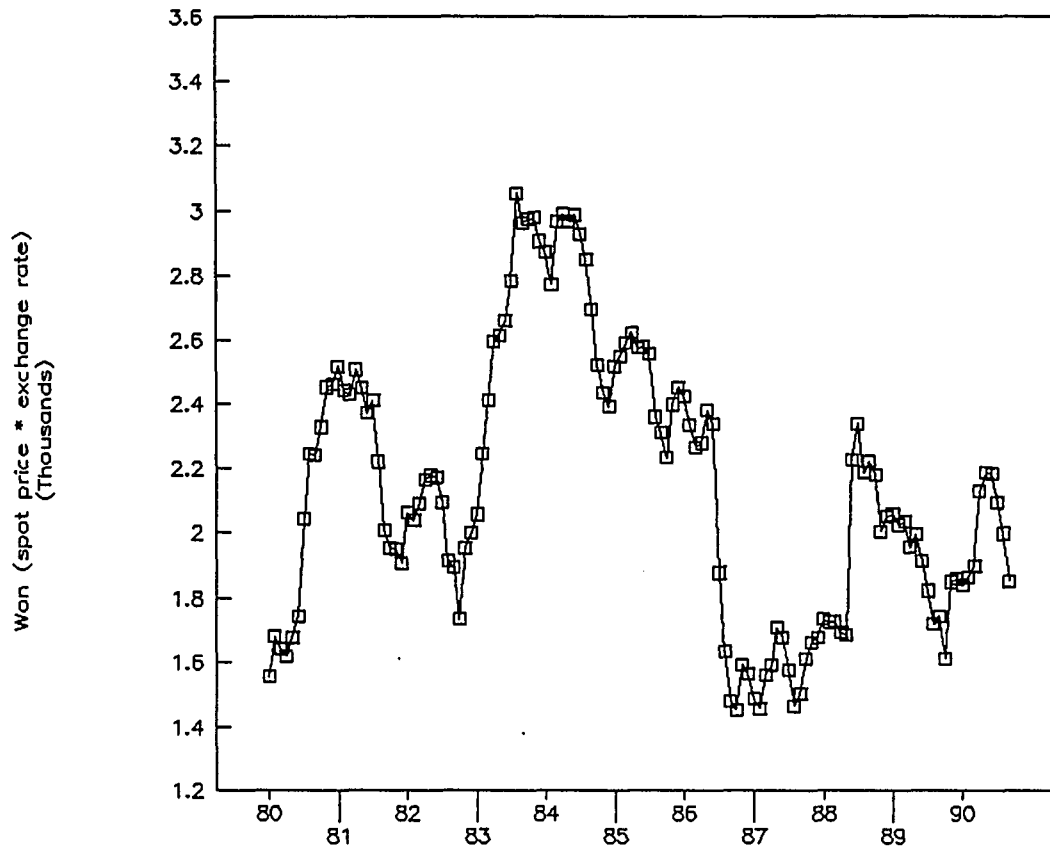


Figure 1. Import price of corn (in terms of Won)

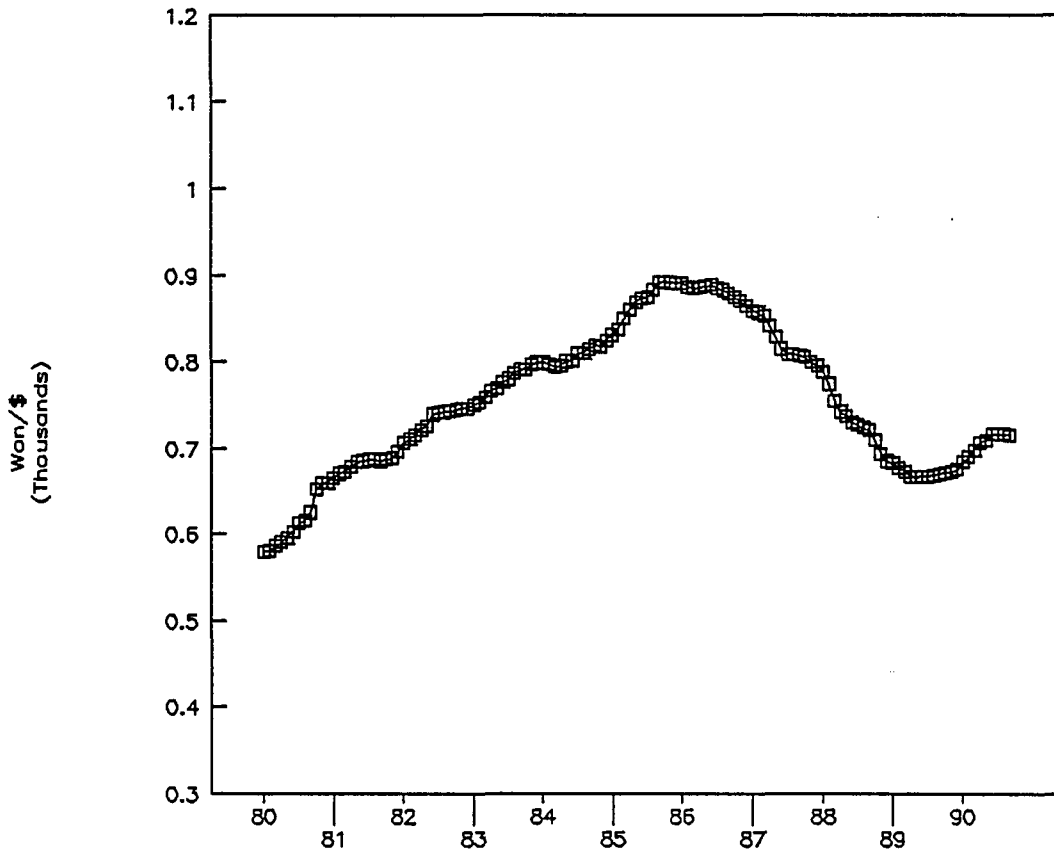


Figure 2. Exchange rate (Korean Won / US Dollar)

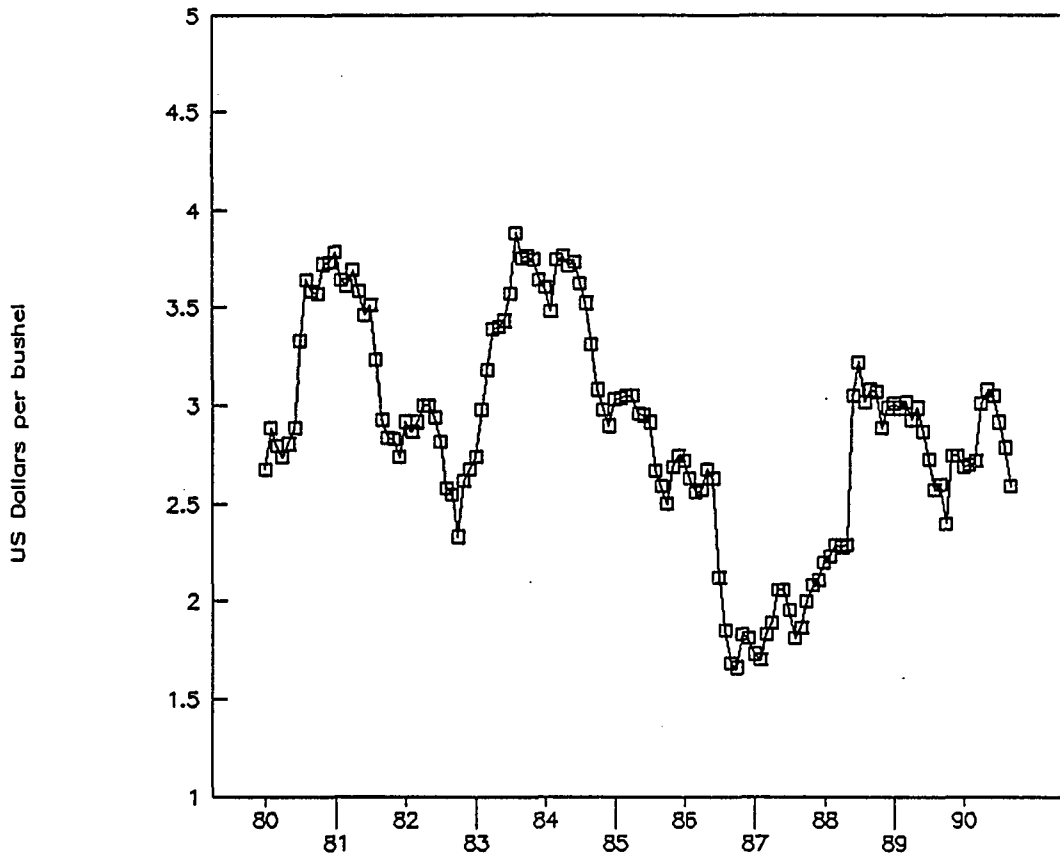


Figure 3. Corn spot price

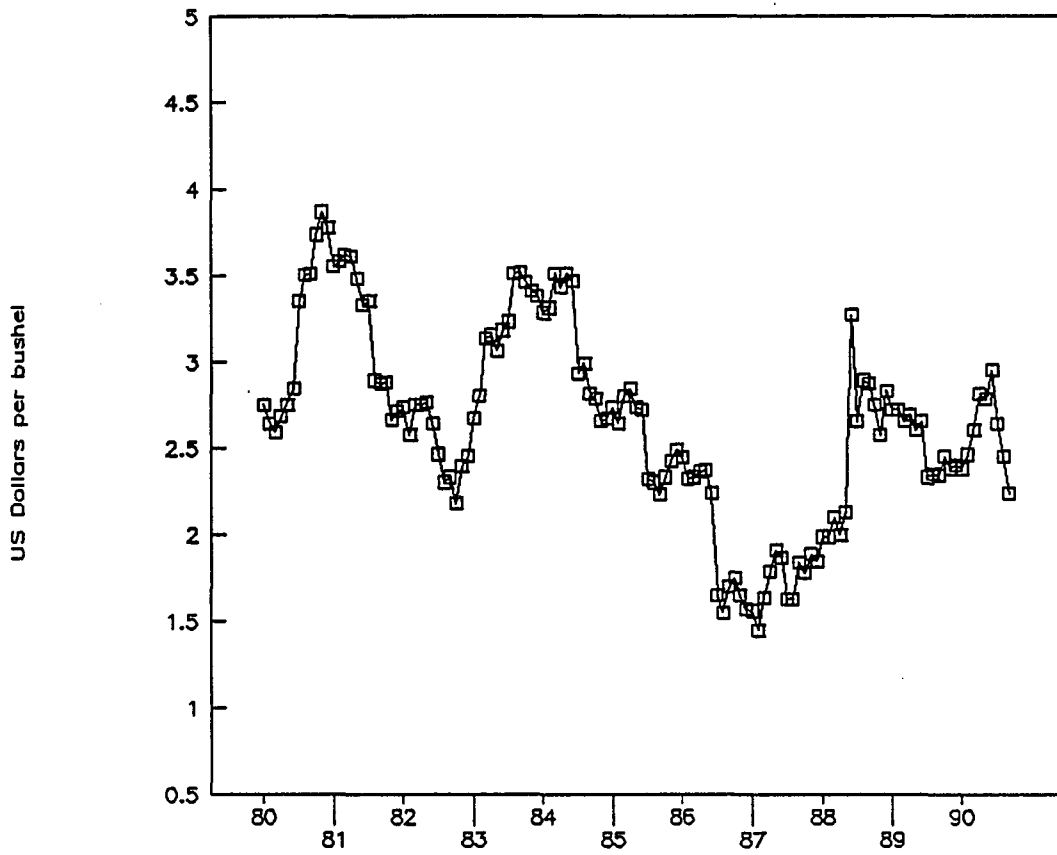


Figure 4. Corn nearby futures price

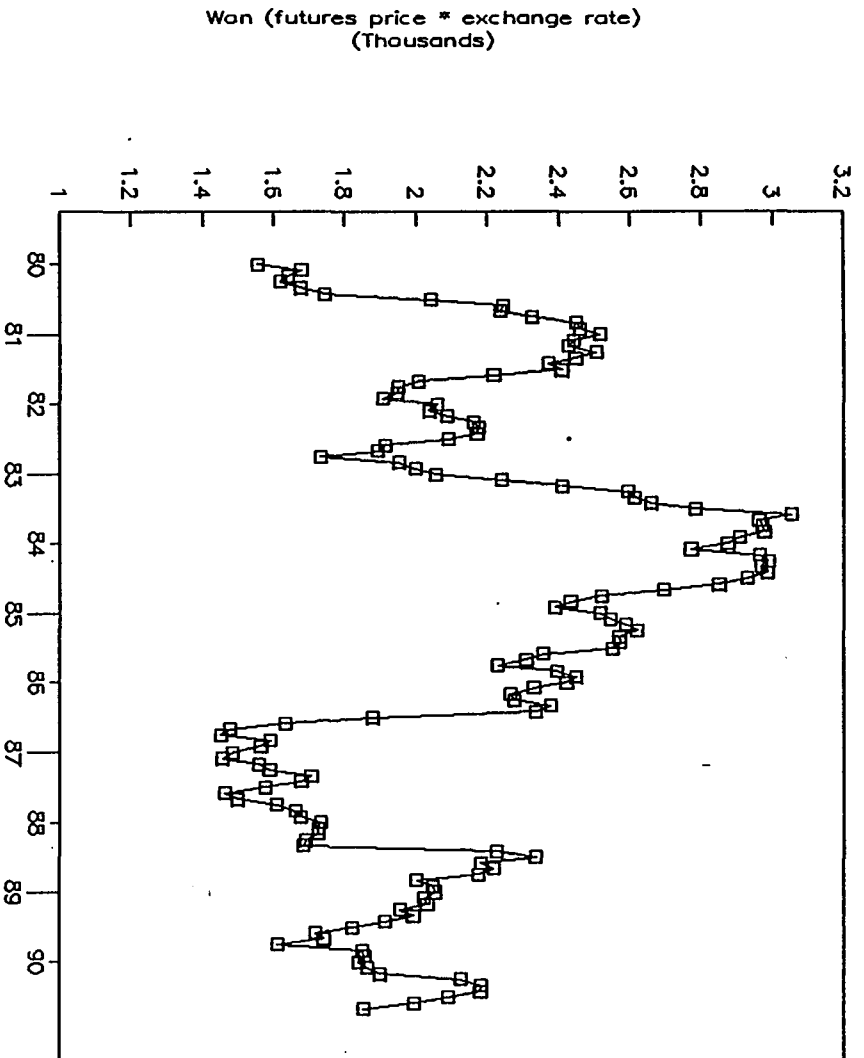


Figure 5. Futures price of corn (in terms of Won)